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Time Use and the Efficiency of Heterogeneous Markups*

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Abstract

What are the welfare implications of markup heterogeneity across firms? In standard monopolistic competition models, such heterogeneity implies inefficiency even in the presence of free entry. We enrich the standard model with heterogeneous firms so that preferences are non-separable in off-market time and market consumption and show that this changes the welfare implications of markup heterogeneity. In this context, homogeneity of markups is neither necessary nor sufficient for efficiency. The marginal cost of the marginal firm is weakly inefficiently high when off-market time and market consumption are complements and inefficiently low when they are substitutes, and the equilibrium allocation devotes weakly too few resources to firm creation. However, when off-market time and market consumption are perfect complements, markups are heterogeneous across firms and yet the equilibrium allocation is efficient.

Keywords: monopolistic competition, markups, efficiency, time use, home production, elasticity of substitution, selection, heterogeneous firms.

JEL codes: D1, D4, D6, L1

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1 Introduction

Studies of markups find large heterogeneity across firms.[1] Does such heterogeneity imply inefficiency? In standard models of monopolistic competition with endogenous entry, such as Dixit and Stiglitz (1977) and Dhingra and Morrow (2019), consumers care solely about varieties of consumption goods and heterogeneous markups imply that the equilibrium allocation is not Pareto efficient. We revisit the analysis of optimal product variety in an environment in which the consumer problem is enriched to allow for market consumption and off-market time in the spirit of Becker (1965). Preferences exhibit constant elasticity of substitution (CES) across varieties of consumption "experiences" that are produced with a home production function that depends non-separably on market consumption and off-market time. We then ask, can equilibrium allocations with heterogeneous markups be efficient in this environment? And, how are markup heterogeneity and firm selection related to the elasticities of substitution across consumption varieties and between off-market time and market consumption?

We show that in this environment constant markups are neither necessary nor sufficient to ensure that allocations are efficient when firms differ in marginal costs. To understand our results, it is instructive to distinguish between two different notions of "markups" generated by our model. The first, which we refer to as "posted markups," is the usual definition of a markup as the difference between the market price of a good and its marginal cost of production, expressed as a fraction of the latter quantity. Due to the presence of home production, the price charged by the firm does not represent the cost of a consumption experience faced by the consumer, because market consumption is only an input into consumption experiences. We therefore contrast the usual markup with an alternative, "holistic" markup that represents the extent to which the total price of an experience deviates from its total marginal cost of production, where both account for the value of the consumer's time. We show that the holistic markup is always smaller than the posted markup and that the two quantities do not necessarily vary in the same way with marginal costs, implying different levels of dispersion.

The presence of home production affects both the positive and normative implications of markups. The fact that preferences over final consumption are CES implies that without home production, markups would be constant across firms of different

[1] See, e.g., Epifani and Gancia (2011), De Loecker et al. (2020), Peters (2020), and Edmond et al. (2023).

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productivity and the first-best outcome would be achieved (see Dhingra and Morrow (2019)). However, the presence of home production changes both the magnitude of the markup and the extent to which it varies with marginal costs. Intuitively, consumers can respond to an increase in the price of a variety by reducing their demand for the variety or by reallocating their time toward home production. We show that whether posted markups increase or decrease in firm productivity depends solely on the elasticity of substitution between off-market time and market consumption: markups rise with productivity and firm size when these are complements and fall when they are substitutes. In the special cases in which the home production function is either Cobb-Douglas or additively separable between off-market time and market consumption, posted markups are constant across firms.

We then turn to an analysis of the distribution of firms that operate in both the efficient and the equilibrium allocations. Because home production affects demand elasticities and firm profits, it alters the incentives for firms to enter and operate and therefore also affects the equilibrium distributions of productivity and firm size. Following Dhingra and Morrow (2019), we model firm entry as a two-step process. First, ex-ante identical firms must pay a fixed cost in order to draw a parameter specifying their marginal cost of production. Second, upon learning their marginal cost, firms must pay a second fixed cost in order to operate. There are therefore two quantities related to firm entry: the mass of firms choosing to draw a productivity parameter, and the fraction of such firms that produce, which is determined by the productivity of the marginal firm. We use the model to study whether the equilibrium values of these quantities are higher or lower than those chosen by a utilitarian planner.

As with the analysis of markups, we find that whether or not the marginal cost of the marginal firm is too high or too low depends solely on the elasticity of substitution between off-market time and market consumption. In equilibrium, the marginal cost of the marginal firm is (weakly) inefficiently high when off-market time and market consumption are complements, and inefficiently low when they are substitutes. However, in the special cases in which off-market time and market consumption are perfect complements or exhibit unit elasticity of substitution, the marginal cost of the marginal firm coincides with the efficient value. Introducing home production therefore has an ambiguous effect on the productivity distribution that arises in equilibrium and its relationship with the efficient allocation.

Having characterized the distribution of firms that operate in efficient and equilibrium allocations, we lastly turn to the aggregate resources devoted to firm entry.

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We show that the equilibrium allocation always devotes (weakly) too few resources to firm entry and product creation, regardless of the elasticity of substitution between off-market time and market consumption. Intuitively, the fact that consumers can substitute off-market time for market consumption implies that the profits of operating firms are "too low" to generate sufficient incentives for entry. However, the sole exception occurs in the case of perfect complements, in which the ratio of off-market time to market consumption is fixed independently of prices and marginal costs. In this case, the mass of firms also coincides with the efficient value, and so the equilibrium allocation in this case is first-best.

The results in this paper have several important implications. First, our result that efficiency obtains in the perfect complements case challenges the idea that variable markups always imply welfare losses.[2] This is potentially important given that estimates of markup dispersion suggest an increase over the last 40 years.[3] Second, the fact that holistic markups are always less than posted markups implies that an exclusive focus on the latter may overstate both the welfare implications of markups and market concentration.

Related literature. The seminal contribution to the theory of optimal product variety is Dixit and Stiglitz (1977), who consider an environment with homogeneous firms and endogenous entry and show that the efficiency of the equilibrium depends on whether preferences over varieties exhibit constant or variable elasticities of substitution. The literature emanating from Dixit and Stiglitz (1977) is vast and so we highlight only closely related developments.[4] Zhelobodko et al. (2012) provide further insight into the VES equilibrium with homogeneous firms and derive general comparative statics. Dhingra and Morrow (2019) extend the analysis of optimal product variety to models with heterogeneous firms, while Behrens et al. (2020) allow for different elasticities between sectors and quantify the welfare losses associated with markups. In these papers, time use is inelastic and CES preferences are both necessary and sufficient for markups to be constant and for allocations to be efficient when marginal costs differ across firms. Our results are reminiscent of Parenti et al. (2017), who enrich the problem of the consumer to incorporate uncertainty over love-for-variety, and also show that variable markups can be consistent with efficiency.

Our modeling of the consumer problem reflects the fundamental idea first dis-

[2]In models such as Peters (2020) and Edmond et al. (2023), markup dispersion creates misallocation and ultimately reduces welfare relative to a benchmark with no dispersion.

[3]See, e.g., Figure 3 of De Loecker et al. (2020) and Figure 2 of Flynn et al. (2019).

[4]See the special issue in Etro (2017) for further discussion and historical context.

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cussed in Becker (1965) and analyzed in depth in Ghez and Becker (1974) that in order to enjoy consumption, consumers must allocate time toward doing so. We follow the literature and refer to this process as "home production," and we use the term "experiences" for the utility-deriving activities produced by combining inputs of time and market consumption. This approach is motivated by empirical evidence that time use and consumption are not separable in preferences.[5] The literature has recently begun to consider the implications of incorporating non-separable preferences for consumption and leisure time for outcomes only indirectly related to consumer time use itself. For example, Boerma and Karabarbounis (2021) and Pretnar (2022) use models with consumption and time-use non-separabilities to measure welfare inequality, Bridgman et al. (2018) and Bednar and Pretnar (2023) study how accounting for time use affects implications for structural change, and Bridgman (2016) studies how in-home productivities for different types of market products have changed over time. We add to these findings by studying the welfare implications of consumption and time-use non-separabilities in an economy with monopolistically competitive firms.

Our incorporation of off-market time into the consumer problem is similar to allowing for elastic labor supply. The literature on monopolistic competition has considered elastic labor, but to the best of our knowledge it has assumed separability from consumption. Bilbiie et al. (2012) explore how the allocation of labor across sectors and the number of products and producers varies over the business cycle. Bilbiie et al. (2019) consider the role that elastic factors of production (labor and capital) play in amplifying distortions on a dynamic path along which firms may enter and exit. Boar and Midrigan (2019) allow for additively separable consumption and time, but focus on how markup distortions redistribute income from laborers to entrepreneurs. Finally, Edmond et al. (2023) disentangle the degree to which markups, misallocation of factors of production, and inefficient entry contribute to welfare costs, and quantify the welfare effects of markups.

The outline of the paper is as follows. Section 2 describes the physical environment and formally defines both the planner's problem and the monopolistically competitive equilibrium. Section 3 characterizes the efficient and equilibrium allocations. Section 4 discusses the relationship between our results and models with no off-market time while characterizing conditions under which equilibrium allocations are efficient. Section 5 concludes.

[5]See, e.g., Aguiar and Hurst (2005), Aguiar and Hurst (2007), Pretnar (2022), and Fang et al. (2022).

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2 Model

We first describe the physical environment and preferences of consumers before turning to formal definitions of the efficient allocation and the monopolistically competitive equilibrium with free entry.

2.1 Physical environment

In this section, we specify the preferences of agents, the technology available to firms, and the aggregate resources in the economy. Subsequent sections characterize efficient allocations and impose a particular market structure.

Consumers. There exists a unit mass of identical consumers who have preferences over a continuum of consumption *experiences*, indexed by a set $i \in \mathcal{I} \subseteq \mathbb{R}$. We denote consumption experiences by c_i. A consumption experience is defined as the act of combining goods and services purchased on the market with off-market time in order to generate final utility. Consumers produce these experiences using a Becker (1965) home production function that takes market consumption, q_i, and off-market time, n_i, as inputs.[6] Each q_i is a particular good or type of good produced by a firm.[7] We assume that the home production function is constant elasticity of substitution (CES) in market quantities and off-market time, so that

$$c_i := c(q_i, n_i) = (\alpha q_i^{\xi} + (1-\alpha)(\zeta n_i)^{\xi})^{1/\xi}, \tag{2.1}$$

where $\alpha \in [0,1]$ is the weight of market consumption in home production, $\zeta \geq 0$ is proportional to the in-home value of off-market time, and the elasticity of substitution between a particular market purchase and its associated allocation of off-market time is governed by $\xi \leq 1$. Note that this object is distinct from the elasticities of substitution across consumption experiences. When market consumption and consumption experiences coincide, the latter is an important object of study in models of monopolistic competition, such as Zhelobodko et al. (2012) and Dhingra and Morrow (2019), who show, to varying degrees, that markups are efficient if and only if the elasticity of substitution for consumption is constant.

[6] Our *experiences* are analogous to what Aguiar et al. (2012), Aguiar and Hurst (2013), and Aguiar and Hurst (2016) refer to as "commodities," which generate utility and require market inputs and time. We sometimes refer to these c_i's as "final consumption."

[7] In the language of Dhingra and Morrow (2019), each q_i corresponds to a particular "variety." We first index quantities by the dummy variable i, and later index them by marginal cost, κ.

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Preferences over consumption experiences are represented by the function,

$$\mathcal{C} = \int_{\mathcal{I}} c_i^{\rho} \mathrm{d}i \tag{2.2}$$

for some parameter ρ governing the substitutability of consumption experiences satisfying $\max\{0, \xi\} \leq \rho < 1$. The specifications (2.1) and (2.2) imply that time use and market consumption are non-separable in preferences. This is the key innovation of our paper relative to the literature on monopolistic competition.[8] When $\alpha = 1$, consumption experiences coincide with market consumption and the specification of preferences in (2.2) is nested within the framework of Dhingra and Morrow (2019). In this formulation, consumers' love-for-variety is, more specifically, a love for a variety of consumption experiences, not just the goods and services that can be bought and sold on the market. That is, consumers want to engage in a variety of different activities with the goods and services they buy, and for that they have a love-for-variety.

Every consumer is endowed with $\overline{T}$ units of efficiency time. In addition to their home production, consumers supply two types of labor to firms: 1) labor devoted to setting up firms (i.e., paying the fixed entry costs); and 2) labor devoted to the production of market consumption by firms. Let $\mathcal{E}$ denote the total amount of labor devoted to start-up costs and $\mathcal{L}$ denote the total amount of labor devoted to market production. If $\mathcal{N} := \int_{\mathcal{I}} n_i \, \mathrm{d}i$ denotes the total amount of time spent engaged in home production then the consumer's time-use constraint is

$$\mathcal{E} + \mathcal{L} + \mathcal{N} \leq \overline{T}. \tag{2.3}$$

Firms. For the modeling of firm technology and entry costs, we deliberately follow Dhingra and Morrow (2019) in order to highlight the novel role played by off-market time. There is a continuum of potential firms that can each produce a unique variety. Firms are ex-ante identical and must pay a fixed cost f_e in order to exist and draw a marginal cost κ from some distribution G. We consider two separate cases for this distribution. In the *homogeneous firms* case, we assume that G is a point mass at some $\overline{\kappa} > 0$, while in the *heterogeneous firms* case, we assume that G has a continuous, positive density, denoted g, that is defined on an interval of the form $[\underline{\kappa}, \infty)$ for some minimal cost $\underline{\kappa} > 0$. Upon drawing their marginal cost, the

[8] Bilbiie et al. (2012) and Bilbiie et al. (2019) feature variable labor supply and leisure that are separable from market consumption, while in Zhelobodko et al. (2012) and Dhingra and Morrow (2019) time use is inelastic and does not enter into an agent's utility function.

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firms must pay an additional fixed cost f in order to remain in business. Both of these fixed costs are denominated in units of effective labor and are interpreted as real resource costs associated with setting up and maintaining a firm (i.e., they are not costs imposed by, e.g., regulation).

We index firms by their marginal cost κ and refer to this as the firm's "type." The output of a firm of type κ that employs ℓ units of effective labor is

$$y(\kappa, \ell) = \ell/\kappa \tag{2.4}$$

so that κ is the marginal cost of the firm and $1/\kappa$ is the firm's labor productivity.

Aggregate resources. Because consumers are identical and labor is the only input used in both production and in setting up firms, the individual time-use constraint is also the aggregate resource constraint. Denoting by M_e the mass of firms that enter and draw a marginal cost, the total effective labor used in production is $\mathcal{L} = \int_{\underline{\kappa}}^{\overline{\kappa}} \ell(\kappa) M_e G(d\kappa)$. Total fixed costs paid by firms are then given by $\mathcal{E} = (f_e + fG(\overline{\kappa}))M_e$. To relate output and costs to this resource constraint, we impose the following assumption on the uniqueness of productive outputs in the set of consumption experiences.

Assumption 1. *The experience set, $\mathcal{I}$, is identical to the production set, $[\underline{\kappa}, \overline{\kappa}]$, so that each q_i is associated with one and only one firm output, $q(\kappa) = y(\kappa, \ell)$.*

Assumption 1 states that each firm produces one and only one type of consumption good, with the quantity denoted $q(\kappa)$, and consumers supply off-market time, $n(\kappa)$, toward engaging in consumption activities associated with that particular type-κ consumption good. This implies that we can index consumer preferences over varieties by $\kappa \in [\underline{\kappa}, \overline{\kappa}]$, and so we can then write $c(\kappa) = c(q(\kappa), n(\kappa))$, and

$$\mathcal{C} = \int_{\underline{\kappa}}^{\overline{\kappa}} c(\kappa)^{\rho} M_e G(d\kappa). \tag{2.5}$$

Labor inputs to production can be written $\ell(\kappa) = \kappa q(\kappa)$, and so the economy-wide resource constraint can then be written

$$(f_e + fG(\overline{\kappa}))M_e + \int_{\underline{\kappa}}^{\overline{\kappa}} (\kappa q(\kappa) + n(\kappa)) M_e G(d\kappa) \leq \overline{T}. \tag{2.6}$$

The following is then the formal definition of the planner's problem.

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Definition 2.1 (Planner's problem)**.** *The problem of the planner is to choose a value of* $\overline{\kappa} > \underline{\kappa}$*, a mass of firms* $M_e > 0$*, and functions* $q(\cdot)$ *and* $n(\cdot)$ *for market consumption and off-market time use to maximize* (2.5) *subject to the resource constraint* (2.6)*.*

The above describes the physical environment and the planner's problem. In what follows we will refer to the solution to the planner's problem in Definition 2.1 as the *efficient* allocation. Note that there are no prices or government transfers in the above, because we have yet to describe the nature of trade between agents.

2.2 Monopolistically competitive equilibrium

We now define the monopolistically competitive equilibrium with free entry. Consumers take prices as given in both product and labor markets, while firms behave in a monopolistically competitive fashion when setting their prices, in the sense that they do not behave strategically but do internalize the effect of their own price on consumer demand. Firms will pay the first fixed cost if doing so gives them non-negative expected profits, and will pay the second fixed cost if doing so provides them with non-negative net operating profits.

Consumers. A consumer earns labor income from supplying time toward setting-up firms, $\mathcal{E}$, and variable hours toward production, $\mathcal{L}$. Hourly wages are normalized to unity. Consumers collectively own the firms in the economy, and so in principle may earn profits $\overline{\Pi}$ net of fixed costs. After invoking Assumption 1 the consumer's budget constraint is

$$\int_{\underline{\kappa}}^{\overline{\kappa}} p(\kappa)q(\kappa)M_e G(d\kappa) = \mathcal{E} + \mathcal{L} + \overline{\Pi} \tag{2.7}$$

where the left-hand side is total expenditure and $p(\kappa)$ are prices for the market-traded goods $q(\kappa)$. Because the model is expressed in terms of efficiency units of time, each $p(\kappa)$ is the price of market-traded goods $q(\kappa)$ relative to the value of off-market time, which is equal to the hourly wage and normalized to unity. Combining (2.7) with (2.3) we get the Beckerian budget constraint:

$$\int_{\underline{\kappa}}^{\overline{\kappa}} (p(\kappa)q(\kappa) + n(\kappa))M_e G(d\kappa) = \overline{T} + \overline{\Pi}. \tag{2.8}$$

The fact that hours devoted to home production cannot be devoted to earning income implies that the total economic cost of a variety κ exceeds the price $p(\kappa)$ in the budget constraint (2.8). We now formally define the consumer's problem.

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Definition 2.2 (Consumer's problem)**.** *The problem of a consumer receiving profits* $\overline{\Pi}$ *and facing a continuum of goods indexed by* $[\underline{\kappa}, \overline{\kappa}]$*, a mass of firms* M_e*, and prices given by the function* p*, is to maximize their utility* (2.5) *subject to the Beckerian budget constraint* (2.8)*.*

Firms. Each firm chooses its own price, taking the prices of all other firms as given. We denote the demand functions for market consumption implied by Definition 2.2 by $q(p; \mathbf{p})$, for a firm that chooses its price p (a scalar) given the function $\mathbf{p}$ of all other prices, although we will omit the second argument when no confusion will arise.[9] The problem of a firm that has paid both the fixed cost f and the entry cost f_e is then described in the following definition.

Definition 2.3 (Firm's problem)**.** *Given the prices* $\mathbf{p}$ *chosen by other firms, the problem of a firm of type* κ *that has paid both fixed costs is*

$$\pi(\kappa; \mathbf{p}) := \max_{p \geq 0} (p - \kappa) q(p; \mathbf{p}). \tag{2.9}$$

The quantity $\pi(\kappa; \mathbf{p})$ *is the operating (or ex-post) profits of a firm of type* κ*.*

In what follows we will omit the dependence of the profits on the prices of other firms and write $\pi(\kappa)$ for the profits of a firm of type κ. A firm that has paid the entry cost, f_e, will pay the second operating cost, f, if and only if $\pi(\kappa) \geq f$. We then have two separate conditions for firm entry, depending on whether we are in the homogeneous firms case or in the heterogeneous firms case. In the homogeneous firms case every firm that pays f_e draws the same $\overline{\kappa}$ and chooses to operate. In the heterogeneous firms case we denote by $\overline{\kappa}$ the solution to $\pi(\overline{\kappa}) = f$, which characterizes the firm that is indifferent between operating and shutting down. Firms will be indifferent to entering if and only if ex-ante (expected) profits net of fixed costs are zero. In general, these ex-ante aggregate profits $\overline{\Pi}$ are given by

$$\overline{\Pi} := M_e \left(\int_{\underline{\kappa}}^{\overline{\kappa}} \pi(\kappa) G(d\kappa) - f_e - f G(\overline{\kappa}) \right) \tag{2.10}$$

and we have the following definition of equilibrium.

Definition 2.4 (Equilibrium)**.** *A monopolistically competitive equilibrium consists of a mass of firms* M_e*, a cutoff value for the marginal cost* $\overline{\kappa}$*, market quantities and*

[9] Due to the assumption that all varieties enter symmetrically into the utility function, there is no need to index the demand functions by κ.

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off-market time $(q, n) = (q(\kappa), n(\kappa))_{\kappa \in [\underline{\kappa}, \overline{\kappa}]}$*, and prices* $p = (p(\kappa))_{\kappa \in [\underline{\kappa}, \overline{\kappa}]}$*, such that the following hold:*

(i) given the cutoff $\overline{\kappa}$ *and prices* p*, the market quantities* q *and off-market time* n *solve the consumer's problem in Definition 2.2;*

(ii) given prices p*, mass of firms* M_e*, and cutoff* $\overline{\kappa}$*, for each* $\kappa \in [\underline{\kappa}, \overline{\kappa}]$*,* $p(\kappa)$ *solves the firm's problem in Definition 2.3;*

(iii) a firm of type κ *produces if and only if* $\pi(\kappa) \geq f$*;*

(iv) aggregate profits in (2.10) are zero, $\overline{\Pi} = 0$*.*

Remark 1 (Homogeneous firms). With homogeneous firms, a non-negligible mass of firms could, in principle, be indifferent between producing and not producing after paying the first entry cost. However, if we write η for the fraction of entering firms that operate, then expected profits are $\overline{\Pi} := M_e(\eta\pi(\overline{\kappa}) - (f_e + f\eta))$. Rearranging this expression and setting to zero then gives $\eta(\pi(\overline{\kappa}) - f) = f_e$, and so the requirement $\pi(\overline{\kappa}) \geq f$ must be strict and hence $\eta = 1$. Intuitively, when productivity is known ex-ante, the decision to operate is redundant given that firms have chosen to enter.

Remark 2 (Special cases). Note that when $\rho = \xi$ preferences coincide with a special case of Bilbiie et al. (2019) in which consumption exhibits CES, and time use and consumption are additively separable. The $\alpha = 1$ case coincides with the CES version of Dhingra and Morrow (2019). Finally, when production is Leontief, the ratio n/q is independent of marginal cost or prices. We will later see that this fact implies that the Leontief case of our model amounts to a disguised form of the CES environment of Dhingra and Morrow (2019) in which effective costs are not κ but instead $\kappa + 1/\zeta$.

3 Analysis

We now turn to characterizing and comparing the efficient and monopolistically competitive equilibrium allocations. We will show that accounting for off-market time use in a non-separable fashion affects the relationship between markups and efficiency because final consumption requires not merely making a market purchase but also allocating time toward using that purchase.

We will distinguish between the markups on market quantities (i.e., the q's) with what we call "holistic" markups, which are the markups of the cost of consumption

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experiences faced by the consumer over their total marginal cost. The total price of a consumption experience from the perspective of the consumer is thus a function of the posted price of the market good, the value of the consumer's off-market time, and the degree to which off-market time and market consumption are complementary or substitutable in home production.

In Dhingra and Morrow (2019), where firms are heterogeneous in productivity and consumer time use is inelastic, CES preferences over (market) consumption are both sufficient and necessary for equilibrium allocations to be socially optimal. In our environment the elasticity of substitution across final consumption experiences is constant, but the prices of final experiences are only equivalent to the prices of market goods when consumers supply time inelastically (i.e., $\alpha = 1$).

3.1 Efficient allocations

The problem of the planner in Definition 2.1 may be written

$$\begin{aligned} &\max_{q,n,M_e,\overline{\kappa}} \int_{\underline{\kappa}}^{\overline{\kappa}} (\alpha q(\kappa)^{\xi} + (1-\alpha)(\zeta n(\kappa))^{\xi})^{\rho/\xi} M_e G(d\kappa) \\ &(f_e + fG(\overline{\kappa}))M_e + \int_{\underline{\kappa}}^{\overline{\kappa}} (\kappa q(\kappa) + n(\kappa)) M_e G(d\kappa) \leq \overline{T}. \end{aligned} \tag{3.1}$$

We first solve for off-market time in the above problem in order to reduce the problem to one of choosing consumption experiences directly. Proceeding in this manner will show that the resource cost of final consumption depends not only on the firm's marginal cost of production κ, but also on both the degree to which market consumption and off-market time are complementary, ξ, and the intensity of final consumption in market goods, α. The first-order conditions for n and q combine to give the efficient ratio of off-market time to market goods

$$n(\kappa)/q(\kappa) = \zeta^{\frac{\xi}{1-\xi}} [(1/\alpha - 1)\kappa]^{\frac{1}{1-\xi}}, \tag{3.2}$$

which leads to the following comparative statics.

Lemma 3.1. *The efficient ratio of off-market time use to market consumption is concave in κ for $\xi \in (-\infty, 0)$, convex in κ for $\xi \in (0, \rho]$, and increasing in κ for all $\xi \in (-\infty, \rho]$. When home production is Leontief, the ratio is constant at $1/\zeta$.*

Proof. This is immediate from the expression (3.2). □

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Lemma 3.1 discusses how $n(\kappa)/q(\kappa)$ varies in κ, which will be informative for our later analysis. For low-productivity varieties (high κ) consumers supply relatively more off-market time and engage in relatively less market consumption than for high-productivity varieties. Further, the rate at which the relative provision of time toward home production changes depends on whether off-market time and market consumption are complements or substitutes. When they are complements, the difference between similar high-productivity firms is greater than the same difference between low-productivity firms, while the opposite is true when time and market consumption are substitutes. In this case, as marginal costs rise consumers substitute time for market consumption at increasing rates.

Holistic production costs. Using the expression (3.2) for the efficient ratio of off-market time to market consumption, we can write the total resource cost associated with a consumption experience κ as $\kappa q(\kappa) + n(\kappa) = \psi(\kappa)c(\kappa)$, where

$$\psi(\kappa) = \psi(\kappa; \alpha, \xi) := \frac{\kappa}{\alpha}\Big(\alpha + (1-\alpha)[(1/\alpha - 1)\zeta\kappa]^{\frac{\xi}{1-\xi}}\Big)^{1-1/\xi}. \tag{3.3}$$

The function ψ represents the total per unit resource cost of a consumption experience inclusive of off-market time. It what follows we will refer to this as the "holistic marginal cost." Because it is fundamental to our analysis, we dwell upon the comparative statics and some special cases of this holistic marginal cost.

First, when home production depends only on off-market time, the cost of a consumption experience is simply the inverse of the time use productivity parameter, $\psi(\kappa; 0, \xi) = 1/\zeta$, while when home production depends only on market consumption, this cost is equal to the marginal cost of production, $\psi(\kappa; 1, \xi) = \kappa$. Second, when off-market time and market consumption are perfect complements ($\xi = -\infty$), the home production technology is Leontief, and the holistic marginal cost of an experience becomes $\psi(\kappa) = \kappa + 1/\zeta$ for all κ. Third, by the envelope theorem (or explicit calculation), the efficient ratio of market consumption, $q(\kappa)$, to total consumption experiences, $c(\kappa)$, is

$$q(\kappa)/c(\kappa) = \psi'(\kappa; \alpha, \xi). \tag{3.4}$$

Consequently, in the above special cases we have $\psi'(\kappa; 1, \xi) = \psi'(\kappa; \alpha, -\infty) = 1$ for all κ, α and ξ, and so market consumption and final consumption coincide when the share of off-market time in consumption experiences vanishes or when off-market time and consumption are perfect complements.

Finally, note that the holistic marginal cost function is always increasing in κ but

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that its asymptotic properties depend on the sign of ξ. Specifically, ψ diverges as $\kappa \to \infty$ for $\xi < 0$ but tends to a finite limit as $\kappa \to \infty$ for $\xi > 0$.[10] In the latter case, off-market time and market consumption are substitutes, and the cost of producing a consumption experience is finite even in the absence of market consumption as an input into home production.

We now introduce an elasticity, novel to this paper, which describes how the holistic marginal cost function varies in productivity. This elasticity will be relevant for the analysis of both the efficient and the equilibrium allocations.[11]

Lemma 3.2 (Elasticity of the holistic marginal cost)**.** *The holistic marginal cost function ψ is strictly increasing and concave for all $\xi \leq 1$, with elasticity*

$$\epsilon(\kappa;\psi) = \frac{\kappa\psi'(\kappa)}{\psi(\kappa)} = \frac{1}{1 + (1/\alpha - 1)^{\frac{1}{1-\xi}}[\kappa\zeta]^{\frac{\xi}{1-\xi}}}. \tag{3.5}$$

This elasticity is bounded above by unity and equal to unity if and only if $\alpha = 1$. Further, it is increasing in κ when $\xi \in [-\infty, 0)$, decreasing in κ when $\xi \in (0, \rho]$, and constant and equal to α when $\xi = 0$.

Proof. See Appendix B. □

Note that as long as home production depends on off-market time, the total per-unit cost of a consumption experience will rise more slowly than the per-unit cost of market consumption because off-market time can substitute for the latter in the home production technology. It is therefore intuitive that the cost elasticity is always below unity unless $\alpha = 1$.

Lemma 3.2 also shows that while the monotonicity of the holistic marginal cost function is independent of ξ, the monotonicity of the elasticity is not. Indeed, whether the elasticity of the holistic marginal cost increases or decreases in the technological marginal cost κ depends solely upon whether off-market time and market consumption are complements or substitutes. These observations will be particularly important for later analysis of equilibrium allocations and markups. In models without off-market time, markups affect the cost associated with final consumption one-for-one. In our model, the shape of the function ψ will determine the degree to which a rise in market prices affects the total cost to the consumer of a consumption experience.

[10] Explicit expressions for the asymptotic behavior of ψ are given in Lemma A.1.

[11] We always define the elasticity of a function f at the value x by $\epsilon(x; f) := xf'(x)/f(x)$. That is, we do not adjust the sign of demand elasticities so that they are positive (as is sometimes done) in order to avoid additional notation.

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We can now use the above to write the planner's problem described in (3.1) directly in terms of consumption experiences, $c(\kappa)$,

$$\begin{aligned} &\max_{c,M_e,\overline{\kappa}} \int_{\underline{\kappa}}^{\overline{\kappa}} c(\kappa)^{\rho} M_e G(d\kappa) \\ &(f_e + fG(\overline{\kappa}))M_e + \int_{\underline{\kappa}}^{\overline{\kappa}} \psi(\kappa)c(\kappa)M_e G(d\kappa) \leq \overline{T}. \end{aligned} \tag{3.6}$$

Proposition 3.3 characterizes efficient allocations of consumption experiences $c(\kappa)$, market consumption $q(\kappa)$, the mass of firms M_e, and the cutoff value of $\overline{\kappa}$.

Proposition 3.3 (Efficient allocations)**.** *In the heterogeneous firms case, if*

$$f \int_{\underline{\kappa}}^{\infty} \psi(\kappa)^{-\frac{\rho}{1-\rho}} G(d\kappa) > (f_e + f) \lim_{\kappa \to \infty} \psi(\kappa)^{-\frac{\rho}{1-\rho}} \tag{3.7}$$

then there exists a unique solution to

$$f \times \int_{\underline{\kappa}}^{\overline{\kappa}} \psi(\kappa)^{-\frac{\rho}{1-\rho}} G(d\kappa) = (f_e + fG(\overline{\kappa})) \times \psi(\overline{\kappa})^{-\frac{\rho}{1-\rho}}, \tag{3.8}$$

which gives the optimal cutoff value of the marginal cost. If (3.7) fails, then the optimal cutoff is $\overline{\kappa} = \infty$. For this $\overline{\kappa}$, the mass M_e of entering firms satisfies

$$(f_e + fG(\overline{\kappa}))M_e = (1-\rho)\overline{T}. \tag{3.9}$$

The efficient quantity of final consumption experiences of every variety is

$$c(\kappa) = \frac{\rho \overline{T} \psi(\kappa)^{-\frac{1}{1-\rho}}}{M_e \int_{\underline{\kappa}}^{\overline{\kappa}} \psi(\kappa)^{-\frac{\rho}{1-\rho}} G(d\kappa)} \tag{3.10}$$

while the efficient market consumption of each variety is $q(\kappa) = \psi'(\kappa)c(\kappa)$. In the homogeneous firms case, the mass of entering firms satisfies

$$(f_e + f)M_e = (1-\rho)\overline{T}, \tag{3.11}$$

each consumption experience is $\overline{c} = (f_e + f)/((1/\rho - 1)\psi(\overline{\kappa}))$, and market consumption of each variety is $\overline{q} = \psi'(\overline{\kappa})\overline{c}$.

Proof. See Appendix B. □

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Some discussion around Proposition 3.3 is warranted. First, note that in equation (3.8), the integral $\int_{\underline{\kappa}}^{\overline{\kappa}} \psi(\kappa)^{-\frac{\rho}{1-\rho}} G(d\kappa)$ may be interpreted as an index that describes the aggregate total resource cost of all consumption experiences. The left-hand side of (3.8) is then the total operating cost while the right-hand side describes the cost of entry plus operation for the last firm to enter. In view of the left-hand side of (3.8), it is efficient for society to produce a particular variety only if the final consumption value of the associated fixed costs is less than or equal to the total consumption value of production. The presence of the holistic marginal cost function in the above problem hints at how consumer behavior in home production impacts the entry margin and firm profitability in the monopolistically competitive equilibrium that we study in the next section.

Equation (3.9) shows that in the efficient allocation, the fraction of the labor endowment $\overline{T}$ devoted to setup costs is always equal to $1-\rho$, regardless of the distribution G, costs f and f_e, or the home production parameters ξ and α. Equation (3.10) characterizes the level of final consumption experiences for each variety. Final consumption is proportional to $\psi(\kappa)^{-\frac{1}{1-\rho}}$, while market consumption differs from this by the factor $\psi'(\kappa)$. When $\alpha = 1$ we obtain the familiar constant elasticity of demand with respect to κ equal to $-1/(1-\rho)$.

3.2 Monopolistically competitive allocations

We now turn to the characterization of monopolistically competitive equilibria. We proceed in a manner similar to that in Section 3.1 in order to highlight the similarities and differences between the efficient and equilibrium allocations. We first characterize the off-market time chosen by the consumer in order to reduce the problem to one in which consumption experiences are the sole object of choice. After this we will turn to an analysis of the pricing and entry decisions of the firms.

Consumer's problem. Given a continuum of firms indexed by the interval $[\underline{\kappa}, \overline{\kappa}]$ and a pricing schedule p, the consumer's problem is to choose market consumption and off-market time $(q, n) = (q(\kappa), n(\kappa))_{\kappa \in [\underline{\kappa}, \overline{\kappa}]}$ satisfying

$$V(p) = \max_{q,n} \int_{\underline{\kappa}}^{\overline{\kappa}} \left(\alpha q(\kappa)^{\xi} + (1-\alpha)(\zeta n(\kappa))^{\xi}\right)^{\rho/\xi} M_e G(d\kappa)$$
$$\int_{\underline{\kappa}}^{\overline{\kappa}} (p(\kappa)q(\kappa) + n(\kappa)) M_e G(d\kappa) = \overline{T} + \overline{\Pi}.$$

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Taking first-order conditions for consumption and time and rearranging provides us with an analog of equation (3.2) for the decentralized economy,

$$n(\kappa)/q(\kappa) = \zeta^{\frac{\xi}{1-\xi}}[(1/\alpha - 1)p(\kappa)]^{\frac{1}{1-\xi}}. \tag{3.12}$$

Substituting the ratio (3.12) into the consumer's problem gives

$$\begin{aligned} V(p) &= \max_{c} \int_{\underline{\kappa}}^{\overline{\kappa}} c(\kappa)^{\rho} M_e G(d\kappa) \\ \int_{\underline{\kappa}}^{\overline{\kappa}} &\psi(p(\kappa))c(\kappa)M_e G(d\kappa) = \overline{T} + \overline{\Pi}. \end{aligned} \tag{3.13}$$

The consumer's problem in (3.13) is similar to the planner's problem in (3.6) except for a few key differences. First, the consumer does not choose the mass of entering firms M_e or the cutoff $\overline{\kappa}$. Second, the resources devoted to setting up firms appear nowhere in the consumer's problem. However, the holistic marginal cost function capturing the role of off-market time appears in both the centralized and the decentralized environments. In the planner's problem this function is evaluated at the technological cost κ, while in the equilibrium allocation it is evaluated at the (yet-to-be-determined) price p chosen by the firm.

To characterize the demand of the consumer and the problem of the firm, it is useful to proceed in steps in order to highlight the role of home production. First note that the first-order conditions for the consumer's problem (3.13) give the demand for each variety of a consumption experience as a function of its price,

$$c(p) = (\rho/\lambda)^{\frac{1}{1-\rho}}\psi(p)^{-\frac{1}{1-\rho}},$$

where λ is the multiplier on the budget constraint. The demand for consumption experiences therefore exhibits constant elasticity with respect to its holistic price $\psi(p)$. Using the condition (3.12), it follows that the market consumption demanded from the firm charging price p satisfies $q(p) = \psi'(p)c(p)$, and so

$$q(p) = (\rho/\lambda)^{\frac{1}{1-\rho}}\psi'(p)\psi(p)^{-\frac{1}{1-\rho}}. \tag{3.14}$$

Firm's problem. Given the demand schedule in (3.14), the firm's problem in

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Definition 2.3 can be written

$$\pi(\kappa) = (\rho/\lambda)^{\frac{1}{1-\rho}} \max_{p \geq 0} (p-\kappa)\psi'(p)\psi(p)^{-\frac{1}{1-\rho}} =: (\rho/\lambda)^{\frac{1}{1-\rho}} \widehat{\pi}(\kappa) \tag{3.15}$$

where the second equality defines $\widehat{\pi}(\kappa)$, a function that gives the profits of the firm up to a constant that is independent of the firm's choices. The optimal price of the firm solves the first-order condition, which may be rearranged to obtain

$$p/\kappa = \frac{\epsilon(p;q)}{\epsilon(p;q)+1} \tag{3.16}$$

where $\epsilon(p;q) = pq'(p)/q(p) < -1$ is the price elasticity of demand. The following lemma shows that this price elasticity admits a closed-form expression.

Lemma 3.4 (Demand elasticity)**.** *The price elasticity of demand is*

$$\epsilon(p;q) = -\frac{1}{1-\xi}\left(1 + \frac{(\rho-\xi)}{(1-\rho)}\epsilon(p;\psi)\right). \tag{3.17}$$

This elasticity satisfies $\epsilon(p;q) \geq -(1-\rho)^{-1}$ for all $p > 0$, with equality if and only if $\epsilon(p;\psi) = 1$ or $\xi = \rho$.

Proof. See Appendix C. □

Recall that in the absence of home production, the price elasticity for each variety is equal to $-(1-\rho)^{-1}$ independently of prices. Lemma 3.4 therefore shows that the presence of home production always makes demand less elastic (i.e., decreases the absolute value of the price elasticity) unless preferences are additively separable ($\xi = \rho$). We now provide some intuition for this result.

Because the demand for market consumption and the demand for consumption experiences are related according to the equation $q(p) = c(p)\psi'(p)$, we can write the demand elasticity as the sum

$$\epsilon(p;q) = \epsilon(p;c) + \epsilon(p;\psi') \tag{3.18}$$

and interpret the effect of home production on price elasticities as the sum of two distinct effects. First, the fact that the demand for consumption experiences is pro-

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portional to $\psi(p)^{-\frac{1}{1-\rho}}$ implies that the associated elasticity satisfies

$$\epsilon(p;c) = -\frac{\epsilon(p;\psi)}{1-\rho} \geq -\frac{1}{1-\rho}$$

where the inequality follows from the fact that $\epsilon(p;\psi) \leq 1$. The presence of home production therefore makes the demand for consumption experiences less elastic with respect to prices because the consumer has some capacity to substitute off-market time for market consumption. This is an intuitive consequence of the fact that the total price to the consumer of a consumption experience rises more slowly than the price charged by the firm.

Second, when the price of a good rises, the consumer also alters the ratio of market consumption to consumption experiences by the factor $\psi'(p) = q(p)/c(p)$. That is, for any fixed level of consumption experience, an increase in the price will lead the consumer to substitute away from market consumption and toward off-market time. The contribution of this effect to the price elasticity corresponds to the second term on the right-hand side of (3.18). Because ψ is concave, this term is weakly negative, and so this force opposes the first effect described above and contributes to making demand more elastic.

Lemma 3.4 shows that the first effect is larger than the second effect, and so the absolute value of the price elasticities are below the values that obtain in the absence of home production. Further, the expression (3.17) shows that the price elasticity is an affine function of the elasticity of the holistic marginal cost. The comparative statics with respect to marginal costs therefore follow immediately from the corresponding claims for the cost elasticity given in Lemma 3.2. Specifically, the price elasticity of demand is decreasing in κ when $\xi \in [-\infty, 0)$, increasing in κ when $\xi \in (0, \rho)$, and constant in κ when $\xi = 0$. The comparative statics for markups can then be inferred directly from the firm's first-order condition in equation (3.16). Proposition 3.5 proceeds in this manner to provide general comparative statics in productivity and identifies three special cases in which markups have a closed-form solution.[12]

Proposition 3.5 (Variable and constant markups)**.** *If $\xi \leq \rho$ then the price $p(\kappa)$ chosen by the firm is unique for all κ and increasing in κ. The posted markup ratio $m(\kappa) := p(\kappa)/\kappa$ is weakly decreasing in κ when $\xi \in [-\infty, 0)$ and is weakly increasing*

[12]Terminology regarding markups is not uniform in the literature. For both of the markup notions considered in this paper, we follow Hall (2018) and refer to p/κ as the "markup ratio," with the "markup" then defined as $p/\kappa - 1$.

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in κ *when* $\xi \in (0, \rho]$. *Finally, when* $\xi \in \{-\infty, 0, \rho\}$, *we have:*

(i) (Leontief) For $\xi = -\infty$ *we have* $m(\kappa) - 1 = (1/\rho - 1)(1 + 1/[\zeta\kappa])$.

(ii) (Cobb-Douglas) For $\xi = 0$, *we have* $m(\kappa) - 1 = (1/\rho - 1)/\alpha$.

(iii) (CES + additively separable leisure) For $\xi = \rho$, *we have* $m(\kappa) - 1 = 1/\rho - 1$.

Finally, the markup ratio always weakly exceeds $1/\rho$ *and coincides with* $1/\rho$ *if and only if* $\alpha = 1$ *or* $\xi = \rho$.

Proof. See Appendix C. □

Proposition 3.5 shows that markups are constant when time use and market consumption exhibit unitary elasticity of substitution (i.e., Cobb-Douglas home production) or preferences are additively separable (i.e., $\xi = \rho$). Further, markups decline in per-unit marginal costs when time use and market consumption are complements and rise when they are substitutes. We now provide some intuition for this variation of markups in the case of complements, with the reverse intuition applying to the case of substitutes.

It is important to recall that by equation (3.18), the effect of home production on price elasticities (and hence the effect on markups) may be decomposed into two distinct and opposing effects. The first-order condition of the consumer shows that the demand for a particular consumption experience always exhibits constant elasticity with respect to the holistic price. Lemma 3.2 then implies that in the case of complements, the demand for a particular consumption experience becomes more elastic as prices increase, which, by itself, suggests that markups ought to decrease with marginal costs. However, by condition (3.12), we also know that the substitution away from market consumption and toward time decreases as prices rise, which contributes to reducing the elasticity of demand. The main insight of Lemma 3.4 is that this second effect is always dominated by the first and that the elasticity of demand is affine in the elasticity of the holistic marginal cost. Lemma 3.2 then implies that markups decline with marginal costs in the case of complements and rise in the case of substitutes.

As we noted above, the role of off-market time in home production implies that the price paid by the consumer is not the sole determinant of their demand for a variety. This distinction between prices and total costs implies that it is instructive to distinguish between two different kinds of markup ratios: the familiar "posted" markup ratio (price over marginal cost), and the following, more holistic notion.

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Definition 3.6 (Holistic markup)**.** *The holistic markup ratio for a firm of type κ charging price $p(\kappa)$ is defined to be $\varphi(\kappa) := \psi(p(\kappa))/\psi(\kappa)$.*

Definition 3.6 describes how much the total price of a consumption experience faced by the consumer is marked up over the total cost of production. It is the more relevant measure of the departure from marginal cost pricing engendered by imperfect competition than the usual (posted) markup, because the latter is only the markup of an input into final consumption.

Now, how do holistic markups experienced by consumers compare to firms' posted markups? In other words, if we account both for consumers' market-purchasing decisions and their associated time-allocation decisions toward utilizing market purchases, how are our inferences regarding markups affected? Lemma 3.7 provides an answer, showing that the holistic markups are always (weakly) lower than the posted markups.

Lemma 3.7. *Posted markups always weakly exceed holistic markups and coincide if and only if $\alpha = 1$.*

Proof. See Appendix C. □

The presence of home production therefore implies that posted markups overestimate the extent to which the total prices faced by consumers depart from their total marginal costs. We will later explore the implications of this observation for the relationship between efficiency and markups.

Equilibrium characterization. The above characterized the behavior of an arbitrary firm in partial equilibrium. In order to characterize the monopolistically competitive equilibrium, we now impose the two equilibrium conditions: 1) the firm with the cutoff value of marginal cost $\overline{\kappa}$ is indifferent between producing and not producing; and 2) firms make zero net profits in expectation. The following Proposition 3.8 is the analogue of Proposition 3.3, as it describes the equilibrium mass of entrants, the cutoff level $\overline{\kappa}$, and final consumption for all varieties.

Proposition 3.8 (Equilibrium allocations)**.** *In the heterogeneous firms case there is a unique monopolistically competitive equilibrium, with cutoff marginal cost $\overline{\kappa}$ given by the unique solution to*

$$f \times \int_{\underline{\kappa}}^{\overline{\kappa}} \widehat{\pi}(\kappa) G(d\kappa) = (f_e + fG(\overline{\kappa})) \times \widehat{\pi}(\overline{\kappa}) \tag{3.19}$$

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where $\widehat{\pi}$ is defined in equation (3.15). For this $\overline{\kappa}$, the mass of firms M_e satisfies

$$(f_e + fG(\overline{\kappa}))M_e = \frac{\overline{T}\int_{\underline{\kappa}}^{\overline{\kappa}} \widehat{\pi}(\kappa)G(d\kappa)}{\int_{\underline{\kappa}}^{\overline{\kappa}} \psi(p(\kappa))^{-\frac{\rho}{1-\rho}} G(d\kappa)} \tag{3.20}$$

The equilibrium quantity of final consumption experiences of every variety is

$$c(p(\kappa)) = \frac{\overline{T}\psi(p(\kappa))^{-\frac{1}{1-\rho}}}{M_e \int_{\underline{\kappa}}^{\overline{\kappa}} \psi(p(\kappa))^{-\frac{\rho}{1-\rho}} G(d\kappa)}, \tag{3.21}$$

and $q(p(\kappa)) = \psi'(p(\kappa))c(p(\kappa))$ for market consumption. When firms are homogeneous and possess the same productivity level $\overline{\kappa}$ and thus the same price $\overline{p} = p(\overline{\kappa})$, the mass of firms M_e satisfies

$$(f_e + f)M_e = (1 - \overline{\kappa}/\overline{p})\epsilon(\overline{p}; \psi)\overline{T}, \tag{3.22}$$

each consumption experience is $\overline{c} = (f_e + f)/((1 - \overline{\kappa}/\overline{p})\epsilon(\overline{p}; \psi)\psi(\overline{p}))$, and market consumption of each variety is $\overline{q} = \psi'(\overline{\kappa})\overline{c}$.

Proof. See Appendix C. □

We will now discuss how the equations characterizing firm selection and entry differ between the competitive and efficient allocations. Our points of comparison between Proposition 3.8 and Proposition 3.3 are the equations governing the productivity of the marginal firm, the mass of entering firms, and the quantities produced of each variety. Recall from previous discussion that $\widehat{\pi}(\kappa)$ represents operating profits up to a constant that is independent of variety. Equation (3.8) equates the total operating costs of final consumption associated with type-κ market goods with that of the last entering firm, and equation (3.19) equates the total operating profits of all production with the operating profits of the last firm to enter. Evidently, whether or not efficient selection is achieved in a competitive environment depends on the variation of the functions $\widehat{\pi}(\kappa)$ and $\psi(\kappa)^{-\frac{\rho}{1-\rho}}$ in marginal costs. Comparing equations (3.9) and (3.20) governing the mass of entering firms, note that while a constant fraction of time is devoted to firm entry costs in an efficient allocation, in the equilibrium allocation this quantity depends non-trivially on the functions $\widehat{\pi}(\kappa)$ and $\psi(p(\kappa))^{-\frac{\rho}{1-\rho}}$. In the next section we turn to the derivation of formal comparative statics.

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4 Welfare analysis

Section 3.1 characterizes the efficient allocations chosen by a benevolent planner and Section 3.2 characterizes the allocations that arise in monopolistically competitive equilibria. We now wish to compare and contrast these two. The fact that firm entry is modeled as a two-step process implies that there are margins along which we can make this comparison: the mass of firms that draw a marginal cost and the fraction of these firms that operate. We therefore study two separate (but related) questions in this section. How does the equilibrium distribution of productivity compare with its analogue in the planner's problem? And is society devoting too many or too few resources to firm entry in equilibrium?

Section 4.1 first compares the productivity of the marginal firm in the equilibrium and efficient allocation, which we refer to as the study of firm "selection." We find that the presence of home production has an ambiguous effect on the distribution of firms that operate in equilibrium and the relationship between this distribution and the efficient distribution. Section 4.2 then characterizes the amount of resources devoted to firm entry, $M_e(f_e + fG(\overline{\kappa}))$, in both the efficient and the equilibrium allocations. In this case there is no ambiguity and we show that this quantity is always (weakly) inefficiently low compared with the efficient allocation. Section 4.3 then ties these observations together, discusses the results, and relates them once more to the literature. In particular, we contrast our results with the known observation that in models without off-market time, CES preferences are both necessary and sufficient to ensure that both markups are constant and allocations are efficient.

The key parameter governing our results is again the elasticity of substitution between off-market time and market consumption. As shown in Proposition 3.5, this elasticity dictates the conditions under which markups are constant and how they vary in firm costs. In this section we also show that it is also integral in determining whether selection and concentration are efficient. Table 1 summarizes our findings regarding efficiency and constancy of markups in the special cases we have heretofore considered.[13]

[13]The "yes" entries in the top part of Table 1 indicate whether or not the particular quantity coincides in equilibrium with its efficient counterpart.

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Table 1: Efficiency and Constancy of Markups

		Special Cases for ξ		
Quantity	Description	$-\infty$	0	ρ
$\overline{\kappa}$	Selection	Yes	Yes	No
$M_e(f_e + fG(\overline{\kappa}))$	Entry costs	Yes	No	No
$\mathcal{C}$	Welfare	Yes	No	No
Equilibrium markups constant?		No	Yes	Yes

4.1 Firm selection

We first turn to a comparison of the distribution of marginal costs that arise in the efficient and equilibrium allocations, which we refer to as the study of firm "selection." To this end, recall that the equilibrium cutoff value of the marginal cost is the solution to equation (3.19), while the efficient cutoff value of the marginal cost is the solution to equation (3.8). Defining

$$J(\overline{\kappa}; F) := \int_{\underline{\kappa}}^{\overline{\kappa}} (F(\kappa)/F(\overline{\kappa}))G(d\kappa) \tag{4.1}$$

for an arbitrary continuous and positive function F defined on $[\underline{\kappa}, \infty)$, note that the efficient cutoff is characterized by the solution to the equation

$$J(\overline{\kappa}; F) = f_e/f + G(\overline{\kappa}) \tag{4.2}$$

for the function $F(\kappa) := \psi(\kappa)^{-\frac{\rho}{1-\rho}}$ and the equilibrium cutoff is a solution to the equation (4.2) for the function $F(\kappa) := \widehat{\pi}(\kappa)$. Further, for any decreasing and differentiable function F we have

$$J'(\overline{\kappa}; F) = g(\overline{\kappa}) - \epsilon(\overline{\kappa}; F) \times \frac{J(\overline{\kappa}; F)}{\overline{\kappa}} \geq 0,$$

where the last inequality follows from the non-negativity of $J(\overline{\kappa}; F)$ and the fact that F is decreasing. Because $J(\underline{\kappa}; F) = 0$ for any F, the equilibrium $\overline{\kappa}$ will therefore be inefficiently high if $J(\kappa; \widehat{\pi}) \leq J(\kappa; \psi^{-\frac{\rho}{1-\rho}})$ for all κ and inefficiently low if the reverse inequality holds for all κ. In order to determine whether the equilibrium cutoff $\overline{\kappa}$ is inefficiently high or low, it will therefore suffice to compare the elasticity of $\widehat{\pi}$ with the

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elasticity of $\psi(\kappa)^{-\frac{\rho}{1-\rho}}$. Proceeding in this manner, we may then derive the following result on firm selection.

Proposition 4.1 (Equilibrium and efficient selection)**.** *The equilibrium cutoff value of $\overline{\kappa}$ is inefficiently high if $\xi \in (-\infty, 0)$, inefficiently low if $\xi \in (0, \rho]$, and efficient if $\xi = -\infty, 0$.*

Proof. See Appendix D. □

Proposition 4.1 shows that when time use is elastically supplied and non-separable with market consumption, the link between markup constancy and efficiency is broken. Indeed, Proposition 4.1 and Proposition 3.5 show that constancy of markups is neither necessary nor sufficient for efficient selection. For instance, markups are variable across firms when home production is Leontief and selection is efficient, and so clearly constancy of markups is not necessary. On the other hand, in the additively separable case with $\xi = \rho$, markups are constant across firms but selection is inefficiently low, and so such constancy is also not sufficient. However, note that if selection is efficient then it is the case that holistic markups are constant across firms. Homogeneity of holistic markups therefore remains necessary for efficiency in this environment.

4.2 Aggregate entry costs

Proposition 4.1 characterizes the fraction $G(\overline{\kappa})$ of the mass of entering firms M_e that operate (i.e., pay the second fixed cost f) in both the efficient and the equilibrium allocations. This analysis of firm selection made no reference to the aggregate amount of resources devoted to firm entry, a topic to which we now turn.

Recall that the first entry cost, f_e, is paid by all of the entering firms, while the second entry cost, f, is paid only by the fraction $G(\overline{\kappa})$ that choose to operate. The total resources expended in the process of firm creation are then $(f_e + fG(\overline{\kappa}))M_e$. Proposition 4.2 below shows that the equilibrium value of this quantity is always (weakly) below its efficient counterpart, and coincides in one special case.

Proposition 4.2 (Inefficiently low entry)**.** *The total resources devoted to firm entry, $M_e(f_e + fG(\overline{\kappa}))$, are inefficiently low in equilibrium unless off-market time and market consumption are perfect complements.*

Proof. See Appendix D. □

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Proposition 4.2 shows that outside of the special case of perfect complementarity between off-market time and market consumption, society is devoting too few resources to firm creation. Note also that due to the assumption of free entry (and hence zero ex-ante profits), in equilibrium the quantity $M_e(f_e + fG(\overline{\kappa}))$ coincides with the aggregate ex-post operating profits $M_e \int_{\underline{\kappa}}^{\overline{\kappa}} \pi(\kappa) G(d\kappa)$ of all active firms. Proposition 4.2 therefore shows that the presence of home production ensures that these profits are typically "too small" to provide incentives for efficient firm entry. Intuitively, consumers in this economy do not internalize the fact that their decisions to allocate labor between off-market time and market production affect firms' profits and the incentives to enter. This is an additional margin of adjustment that is not present in models without off-market time.

4.3 Discussion

Proposition 4.1 characterizes when the efficient and equilibrium cutoff values of $\overline{\kappa}$ coincide, while Proposition 4.2 characterizes when the efficient and equilibrium values of $(f_e + fG(\overline{\kappa}))M_e$ coincide. Combining these two results, we see that the efficient and equilibrium allocations cannot possibly coincide if $\xi \neq -\infty$. The following proposition proves the converse.

Proposition 4.3 (Efficiency)**.** *The equilibrium allocation is efficient if and only if off-market time and market consumption are perfect complements.*

Proof. See Appendix D. □

We believe that it is intuitive why the equilibrium allocation is efficient in the case of perfect complements. The basic insight is that this case is essentially a disguised version of the CES case considered in Dhingra and Morrow (2019), in which the technological cost is not κ but instead $\kappa + 1/\zeta$. To understand how this follows from the above analysis, note that under Leontief home production we can simply add and subtract $1/\zeta$ from the profit margin of the firm to obtain

$$p - \kappa = p + 1/\zeta - (\kappa + 1/\zeta) = \psi(p) - \psi(\kappa)$$

and since $\psi'(p) = 1$ in this case, the profits of the firm are proportional to

$$\widehat{\pi}(\kappa) = \max_{\psi(p) \geq 0} (\psi(p) - \psi(\kappa))\psi(p)^{-\frac{1}{1-\rho}}.$$

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Writing the firm's problem in this manner shows that the firm charges a constant markup of the total price over the total marginal cost. Then the results of Dhingra and Morrow (2019) pertaining to efficiency of markups, selection, and concentration, all follow immediately. The key point here is that in the Leontief case, the ratio of off-market time to market consumption is fixed independently of prices and coincides with that used in the efficient allocation. This simple observation is particularly noteworthy because condition (i) of Proposition 3.5 shows that the (posted) markups in this case are not constant across firms. An economist who observed this heterogeneity would therefore erroneously conclude that welfare losses existed in this environment if they ignored the role of home production. Empirical work seems to suggest an increasing relationship between markups and productivities.[14] The above example shows that such variable markups may be efficient if market consumption and off-market time are perfect complements.

Efficiency and demand elasticities. In this paper we have characterized efficient and equilibrium allocations in an environment in which consumers value variety in consumption experiences and have some capacity to substitute off-market time for market consumption. We now wish to relate this once more to the analysis of Dhingra and Morrow (2019), who allow for an arbitrary utility function over consumption but do not allow for off-market time.

Have we simply considered a disguised form of their environment for a different choice of utility? That is, does incorporating off-market time simply change demand elasticities but not the interpretation of these elasticities? In this section we show that the answer to both of these questions is "no." To establish this, we will construct an example of our model that produces the same demand schedules as a particular case of Dhingra and Morrow (2019), but show that while the allocation in their model is inefficient, the associated allocation in our model is efficient.

Consider an environment in which firms are homogeneous and consumers do not value off-market time and have utility function $u(q) = q^\rho - \eta q$ for some $\eta > 0$. The elasticity of utility is then

$$\epsilon(q;u) = \frac{qu'(q)}{u(q)} = \rho - \frac{(1-\rho)\eta}{q^{\rho-1} - \eta} \tag{4.3}$$

and so $\epsilon'(q;u) < 0$. The results of Dixit and Stiglitz (1977) (see also Proposition 3 on page 211 of Dhingra and Morrow (2019)) imply that there is excess entry in

[14] See, e.g., Edmond and Veldkamp (2009); Berry et al. (2019); Peters (2020).

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equilibrium and that the allocation is not efficient. Given a multiplier λ on the budget constraint, the first-order condition for the variety produced by the firm of type κ is $\rho q(\kappa)^{\rho-1} - \eta = \lambda p(\kappa)$, which rearranges to give the demand schedule

$$q(\kappa) = (\lambda/\rho)^{-\frac{1}{1-\rho}}(p(\kappa) + \eta/\lambda)^{-\frac{1}{1-\rho}}, \tag{4.4}$$

with λ and the mass of firms M_e then determined by the zero profits condition and the consumer's budget constraint. The point of this calculation is that the equilibrium demand schedules in the economy with $u(q) = q^\rho - \eta q$ and no home production are identical to those in an economy with Leontief home production and CES preferences over final consumption experiences in which $\zeta = \lambda/\eta$ (although the labor endowment $\overline{T}$ will differ). However, the equilibrium in the economy with time use is efficient, while the equilibrium in the above example economy is not. To resolve this apparent contradiction, note that in the economy with time use, the $1/\zeta$ term appearing in the demand schedule is exogenous and represents the real resource cost associated with a consumer's time. In contrast, the η/λ term appearing in the demand schedule of the economy without time use depends on the endogenous multiplier λ on the budget constraint of the consumer, and can therefore be affected by taxes or subsidies on entry or labor supply.

Dhingra and Morrow (2019) state their results on firm selection in terms of the utility elasticity defined above in (4.3). Because the utility derived from a consumption variety in our model depends on both market consumption and off-market time, there is no direct analogue of their utility elasticity. However, we believe that the above example shows that incorporating time use affects our understanding of optimum product variety in a manner not captured by generalizing the utility function over consumption varieties beyond the CES specification.

Extensions to other environments. The key insight that we have repeatedly emphasized in this paper is that the distribution of markups is not a sufficient statistic for inferring aggregate welfare when market consumption and off-market time enter non-separably into preferences. This feature ensured that consumers must consider the value of their time when making a market purchase, so that the price paid to the firm did not represent the true economic price of the associated variety. In this approach we were motivated by the insights of Becker (1965), which we regard as a natural and concrete enrichment of the consumer problem.

However, the above insight may be more generally applicable to other environ-

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ments. For instance, suppose that consumers care solely about a single consumption good produced by a competitive sector of final goods producers, and that the technology of these producers is a CES function of a continuum of intermediate varieties. Suppose further that the production of each variety depends non-separably on both labor and the output of monopolistically competitive firms taking labor as their sole input. Then the situation would be similar to that analyzed in this paper, insofar as the markups of the intermediate producers would not represent the extent to which the price of a variety differed from its marginal cost of production. Rather, the markup charged by the intermediate producers would represent a markup on an *input* into the variety entering the technology of final producers. Exploring the extent to which the insights of this paper apply in such a setting could be a fruitful area for future research.

5 Conclusion

This paper has presented a parsimonious model that shows how the welfare effects of markups depend upon the extent to which consumers value off-market time. The key insight is that in our model, the usual definition of markups (which we have termed "posted" markups) differs from a more holistic definition that incorporates the fact that the price paid by consumers is not the sole economic cost they incur when consuming a good. For the special case of perfect complements between market consumption and off-market time, we have shown that heterogeneous markups can be consistent with a first-best allocation.

We believe that our work ought to encourage researchers to continue to consider how accounting for different household decision structures affects inferences regarding efficiency when faced with market structures that allow for prices to exceed marginal costs. We have shown that there exists an economy in which variable markups are indeed efficient. The recent work of Parenti et al. (2017) represents a different enrichment of the standard consumer problem in which efficiency is also consistent with heterogeneous markups. It thus remains for researchers to determine which decision structures themselves are most plausible when assessing what such structures imply for the efficiency of environments in which firms operate with imperfect competition.

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A Preliminary results

In this appendix we derive some preliminary results that are used for the proofs of the claims regarding both the efficient and the equilibrium allocations. First recall the definition of the holistic marginal cost of a consumption experience,

$$\psi(\kappa;\alpha,\xi)=\frac{\kappa}{\alpha}\Big(\alpha+(1-\alpha)[(1/\alpha-1)\zeta\kappa]^{\frac{\xi}{1-\xi}}\Big)^{1-1/\xi}. \tag{A.1}$$

We first record some asymptotic properties of the holistic marginal cost function.

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Lemma A.1 (Asymptotic behavior of ψ)**.** *For $\xi \in (0, \rho)$, for large κ we have the finite limit*

$$\lim_{\kappa\to\infty} \psi(\kappa) = (1-\alpha)^{-1/\xi}/\zeta \tag{A.2}$$

and for small $\kappa > 0$ we have the asymptotic relationship

$$\lim_{\kappa\to 0^+} \psi(\kappa)/\kappa = \alpha^{-1/\xi}. \tag{A.3}$$

For $\xi \in [-\infty, 0)$, for large κ we have the asymptotic relationship

$$\lim_{\kappa\to\infty} \psi(\kappa)/\kappa = \alpha^{-1/\xi} \tag{A.4}$$

and for small κ we have

$$\lim_{\kappa\to 0^+} \psi(\kappa) = (1-\alpha)^{-1/\xi}/\zeta. \tag{A.5}$$

Finally, for $\xi = 0$, the total cost function is $\psi(\kappa) = \alpha^{-\alpha}[(1-\alpha)\zeta]^{-(1-\alpha)}\kappa^{\alpha}$.

Proof. For $\xi \in (0, \rho)$ we have the limit

$$\lim_{\kappa\to\infty} \frac{\alpha + (1-\alpha)[(1/\alpha - 1)\zeta\kappa]^{\frac{\xi}{1-\xi}}}{(1-\alpha)[(1/\alpha - 1)\zeta\kappa]^{\frac{\xi}{1-\xi}}} = 1 \tag{A.6}$$

and therefore have

$$\begin{aligned}
\lim_{\kappa\to\infty} \psi(\kappa) &= \lim_{\kappa\to\infty} \frac{\kappa}{\alpha}\Big((1-\alpha)[(1/\alpha - 1)\zeta\kappa]^{\frac{\xi}{1-\xi}}\Big)^{1-1/\xi} \\
&\quad \times \lim_{\kappa\to\infty} \left(\frac{\alpha + (1-\alpha)[(1/\alpha - 1)\zeta\kappa]^{\frac{\xi}{1-\xi}}}{(1-\alpha)[(1/\alpha - 1)\zeta\kappa]^{\frac{\xi}{1-\xi}}}\right)^{1-1/\xi} \\
&= \frac{1}{\alpha}(1-\alpha)^{1-1/\xi}(1/\alpha - 1)^{-1}(1/\zeta) \times 1
\end{aligned}$$

which simplifies to (A.2). For $\xi \in (0, \rho)$ and small $\kappa > 0$ the limit (A.3) follows from direct substitution. For $\xi \in [-\infty, 0)$, for large κ the limit (A.5) follows from direct calculation, while for small κ (and $\xi < 0$) we have the limit

$$\lim_{\kappa\to 0} \frac{\alpha + (1-\alpha)[(1/\alpha - 1)\zeta\kappa]^{\frac{\xi}{1-\xi}}}{(1-\alpha)[(1/\alpha - 1)\zeta\kappa]^{\frac{\xi}{1-\xi}}} = 1 \tag{A.7}$$

which gives (A.5). □

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Lemma A.2. *The derivative of the total cost function in (A.1) is given by*

$$\psi'(\kappa;\alpha,\xi) = \left(\alpha + (1-\alpha)[(1/\alpha - 1)\zeta\kappa]^{\frac{\xi}{1-\xi}}\right)^{-1/\xi}. \tag{A.8}$$

Proof. Explicit calculation gives

$$\begin{aligned}
\psi'(\kappa) &= \frac{1}{\alpha}\left(\alpha + (1-\alpha)[(1/\alpha - 1)\zeta\kappa]^{\frac{\xi}{1-\xi}}\right)^{1-1/\xi} \\
&\quad + (1 - 1/\xi) \times \frac{\xi}{1-\xi} \times (1-\alpha)[(1/\alpha - 1)\zeta\kappa]^{\frac{\xi}{1-\xi}}\kappa^{-1} \\
&\quad \times \frac{\kappa}{\alpha}\left(\alpha + (1-\alpha)[(1/\alpha - 1)\zeta\kappa]^{\frac{\xi}{1-\xi}}\right)^{-1/\xi} \\
&= \frac{1}{\alpha}\left(\alpha + (1-\alpha)[(1/\alpha - 1)\zeta\kappa]^{\frac{\xi}{1-\xi}}\right)^{1-1/\xi} \\
&\quad - (1-\alpha)[(1/\alpha - 1)\zeta\kappa]^{\frac{\xi}{1-\xi}}\frac{1}{\alpha}\left(\alpha + (1-\alpha)[(1/\alpha - 1)\zeta\kappa]^{\frac{\xi}{1-\xi}}\right)^{-1/\xi}
\end{aligned}$$

which simplifies as claimed. □

Lemma A.3 (Special cases for ψ)**.** *When $\xi = -\infty$ or $\xi = 0$, the total cost ψ and marginal cost ψ' become*

$$\begin{aligned}
\psi(\kappa;\alpha,0) &= \alpha^{-\alpha}[(1-\alpha)\zeta]^{-(1-\alpha)}\kappa^{\alpha} \\
\psi'(\kappa;\alpha,0) &= [(1/\alpha - 1)\zeta\kappa]^{-(1-\alpha)} \\
\psi(\kappa;\alpha,-\infty) &= \kappa + 1/\zeta \\
\psi'(\kappa;\alpha,-\infty) &= 1.
\end{aligned} \tag{A.9}$$

Proof. The above calculations all follow by noting that $n = q/\zeta$ in the Leontief case and $n = (1/\alpha - 1)\kappa q$ in the Cobb-Douglas case. □

The following lemma will be used for the characterization of the marginal firm in both the efficient and equilibrium allocations.

Lemma A.4. *For any continuously differentiable and decreasing function $h : [\underline{\kappa}, \infty) \to (0, \infty)$, define the function*

$$H(\kappa; h) := \frac{f\int_{\underline{\kappa}}^{\kappa} h(x)G(dx)}{h(\kappa)(f_e + fG(\kappa))} - 1. \tag{A.10}$$

Then there exists a unique solution $\overline{\kappa}$ to $H(\overline{\kappa}; h) = 0$ if $\liminf_{\overline{\kappa}\to\infty} H(\overline{\kappa}; h) > 0$.

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Proof. Note that $H(\underline{\kappa}; h) = -1$ because the numerator of the quotient vanishes at $\overline{\kappa} = \underline{\kappa}$. Now consider the function $L(\kappa) := f \int_{\underline{\kappa}}^{\kappa} h(\kappa) G(d\kappa) - h(\kappa)(f_e + fG(\kappa))$ and note that for all $\kappa \in [\underline{\kappa}, \infty)$,

$$L'(\kappa) = fh(\kappa)g(\kappa) - fh(\kappa)g(\kappa) - h'(\kappa)(f_e + fG(\kappa)) > 0$$

which establishes uniqueness, because H and L share the same roots. □

Recalling the definition

$$\widehat{\pi}(\kappa; \alpha, \xi) := \max_{p \geq 0} (p - \kappa)\psi'(p; \alpha, \xi)\psi(p; \alpha, \xi)^{-\frac{1}{1-\rho}},$$

we have the following, which will be used in the proof of Proposition 4.1.

Lemma A.5 (Special cases for $\widehat{\pi}$)**.** *When $\xi = 0$, the function $\widehat{\pi}$ becomes*

$$\widehat{\pi}(\kappa; \alpha, 0) = E(\alpha, \zeta, \rho)\kappa^{-\frac{\rho\alpha}{1-\rho}} \tag{A.11}$$

for some constant $E(\alpha, \zeta, \rho)$, while for $\xi = -\infty$ we have

$$\widehat{\pi}(\kappa; \alpha, -\infty) = \rho^{\frac{1}{1-\rho}}(1/\rho - 1)(\kappa + 1/\zeta)^{-\frac{\rho}{1-\rho}}. \tag{A.12}$$

Proof. Omitting arguments for ease of notation, when $\xi = 0$ we have

$$\begin{aligned}\widehat{\pi}(\kappa; \alpha, 0) &= \max_{p \geq 0} (p - \kappa)[(1/\alpha - 1)\zeta p]^{-(1-\alpha)}\left(\alpha^{-\alpha}[(1-\alpha)\zeta]^{-(1-\alpha)}p^{\alpha}\right)^{-\frac{1}{1-\rho}} \\ &= D(\alpha, \zeta, \rho) \max_{p \geq 0} (p - \kappa)p^{-\frac{\rho\alpha}{1-\rho}-1}\end{aligned}$$

where $D(\alpha, \zeta, \rho) := \alpha^{\frac{1}{1-\rho}}[(1/\alpha - 1)\zeta]^{\frac{\rho(1-\alpha)}{1-\rho}}$ is a constant that is irrelevant to the firm's pricing decision. The first-order condition for the price is

$$0 = -\left(\frac{\rho\alpha}{1-\rho} + 1\right)(p - \kappa)p^{-\frac{\rho\alpha}{1-\rho}-2} + p^{-\frac{\rho\alpha}{1-\rho}-1}$$

which then simplifies to $(\rho\alpha/(1-\rho) + 1)(p - \kappa) = p$, and hence

$$p/\kappa = 1/\rho + (1/\rho - 1)(1/\alpha - 1).$$

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Substitution then gives $p - \kappa = [(1/\rho - 1)/\alpha]\kappa$

$$\begin{aligned}\widehat{\pi}(\kappa) &= D(\alpha,\zeta,\rho)[(1/\rho-1)/\alpha]\kappa((1/\rho+(1/\rho-1)(1/\alpha-1))\kappa)^{-\frac{\rho\alpha}{1-\rho}-1}\\ &=: E(\alpha,\zeta,\rho)\kappa^{-\frac{\rho\alpha}{1-\rho}}\end{aligned}$$

for some (again unimportant) constant $E(\alpha,\zeta,\rho)$. For the Leontief case, we have

$$\widehat{\pi}(\kappa;\alpha,-\infty) := \max_{p\geq 0}\ (p-\kappa)(p+1/\zeta)^{-\frac{1}{1-\rho}}.$$

The first-order condition is then

$$0 = -\frac{1}{1-\rho}(p-\kappa)(p+1/\zeta)^{-\frac{1}{1-\rho}-1} + (p+1/\zeta)^{-\frac{1}{1-\rho}}$$

and hence $p + 1/\zeta = (\kappa + 1/\zeta)/\rho$. Substitution then gives the expression for $\widehat{\pi}$. □

B Efficient allocations

We now record the proofs for all claims pertaining to the characterization of efficient allocations.

Proof of Lemma 3.2. Proceeding from the expression for the marginal total cost in Lemma A.2, we have

$$\begin{aligned}\psi''(\kappa) = -\frac{1}{\xi}\Big(\alpha + (1-\alpha)[(1/\alpha-1)\zeta\kappa]^{\frac{\xi}{1-\xi}}\Big)^{-1/\xi-1}\\ \times\frac{\xi}{1-\xi}\times(1-\alpha)[(1/\alpha-1)\zeta\kappa]^{\frac{\xi}{1-\xi}}\times\kappa^{-1} < 0,\end{aligned} \tag{B.1}$$

which establishes concavity. Using the expression for ψ' in Lemma A.2, we have

$$\frac{\psi(\kappa)}{\psi'(\kappa)} = \kappa + (1/\alpha-1)\kappa[(1/\alpha-1)\zeta\kappa]^{\frac{\xi}{1-\xi}}$$

which gives the claimed expression for the elasticity. The remaining claims are obvious by inspection. □

Proof of Proposition 3.3. We will first find the optimal quantities for each consumption experiences for fixed $\overline{\kappa}$ and M_e, before turning to the optimal choices of these

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latter quantities.[15] The first-order conditions for consumption experiences in the planner's problem (3.6) are $\rho c(\kappa)^{\rho-1} = \lambda_p \psi(\kappa)$ or

$$c(\kappa) = (\rho/\lambda_p)^{\frac{1}{1-\rho}} \psi(\kappa)^{-\frac{1}{1-\rho}}. \tag{B.2}$$

We define $I(\overline{\kappa}, M_e) := \overline{T} - (f_e + fG(\overline{\kappa}))M_e$ for the total amount of the endowment net of entry costs. Substituting the expressions (B.2) into the resource constraint $\int_{\underline{\kappa}}^{\overline{\kappa}} \psi(\kappa)c(\kappa)M_e G(d\kappa) = I(\overline{\kappa}, M_e)$ then gives

$$I(\overline{\kappa}, M_e) = (\rho/\lambda_p)^{\frac{1}{1-\rho}} \int_{\underline{\kappa}}^{\overline{\kappa}} \psi(\kappa)^{-\frac{\rho}{1-\rho}} M_e G(d\kappa)$$

and so combining with (B.2) again, we have

$$c(\kappa) = \frac{I(\overline{\kappa}, M_e)\psi(\kappa)^{-\frac{1}{1-\rho}}}{\int_{\underline{\kappa}}^{\overline{\kappa}} \psi(\kappa)^{-\frac{\rho}{1-\rho}} M_e G(d\kappa)}. \tag{B.3}$$

Substitution into the payoff for a particular M_e and $\overline{\kappa}$ then gives the objective

$$W(M_e, \overline{\kappa}) = \int_{\underline{\kappa}}^{\overline{\kappa}} c(\kappa)^\rho M_e G(d\kappa) = I(\overline{\kappa}, M_e)^\rho M_e^{1-\rho} \left(\int_{\underline{\kappa}}^{\overline{\kappa}} \psi(\kappa)^{-\frac{\rho}{1-\rho}} G(d\kappa) \right)^{1-\rho}.$$

Taking logs and dividing by ρ then gives the equivalent maximization problem

$$\begin{aligned} \max_{M_e, \overline{\kappa}} \Bigg\{ & \ln(\overline{T} - (f_e + fG(\overline{\kappa}))M_e) + (1/\rho - 1)\ln M_e \\ & + (1/\rho - 1)\ln\left(\int_{\underline{\kappa}}^{\overline{\kappa}} \psi(\kappa)^{-\frac{\rho}{1-\rho}} G(d\kappa) \right) \Bigg\}. \end{aligned} \tag{B.4}$$

For any fixed $\overline{\kappa}$, the objective in problem (B.4) diverges to $-\infty$ as $M_e \to 0$ from above or $M_e \to \overline{T}/(f_e + fG(\overline{\kappa}))$ from below, and so the optimal M_e may therefore be found by solving the first-order condition with respect to M_e, which implies that the optimal mass of firms satisfies (3.9). When combined again with (B.3), this gives the claimed expressions for final consumption in (3.10).

Finally, using the above analysis, the problem of the planner choosing the cutoff

[15] Dhingra and Morrow (2019) proceed in this manner in the proof of their Proposition 1.

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is equivalent to

$$\max_{\overline{\kappa}>0} \left\{ \ln\left(\int_{\underline{\kappa}}^{\overline{\kappa}} \psi(\kappa)^{-\frac{\rho}{1-\rho}} G(d\kappa) \right) - \ln(f_e + fG(\overline{\kappa})) \right\}.$$

This last objective is not necessarily concave in $\overline{\kappa}$. However, the objective diverges to negative infinity as $\overline{\kappa} \to \underline{\kappa}$ from above, and the derivative in $\overline{\kappa}$ is given by

$$\left(\frac{\psi(\overline{\kappa})^{-\frac{\rho}{1-\rho}}}{\int_{\underline{\kappa}}^{\overline{\kappa}} \psi(\kappa)^{-\frac{\rho}{1-\rho}} G(d\kappa)} - \frac{f}{f_e + fG(\overline{\kappa})} \right) \times g(\overline{\kappa}) \tag{B.5}$$

which is either positive if (3.7) fails (and hence the planner chooses $\overline{\kappa} = \infty$) or negative for large $\overline{\kappa}$. In the latter case, the optimal value of $\overline{\kappa}$ therefore exists and must satisfy the first-order condition (3.8), and applying Lemma A.4 with $h(\kappa) := \psi(\kappa)^{-\frac{\rho}{1-\rho}}$ ensures that there is a unique solution to this last equation. Given the values of $\overline{\kappa}$ and M_e found above, the expressions for market consumption then follow from the expression (B.3) and equation (3.4). Finally, in the case with homogeneous firms we have $G(\overline{\kappa}) = 1$, and the expressions for $\overline{c}$ and $\overline{q}$ are immediate. □

C Equilibrium allocations

We now record the proofs for all claims pertaining to the characterization of the monopolistically competitive equilibrium allocations. We first characterize the price elasticity of demand in terms of the cost elasticity.

Lemma C.1. *The elasticity of the marginal total cost with respect to productivity is given by* $\epsilon(\kappa; \psi') = (\epsilon(\kappa; \psi) - 1)/(1 - \xi)$.

Proof. Using the expression for the derivative of the total cost function given in Lemma A.2 and the expression (B.1) for the second derivative in the proof of Lemma 3.2, we have

$$\begin{aligned} \frac{\kappa\psi''(\kappa)}{\psi'(\kappa)} &= -\frac{1}{1-\xi}\left(\alpha + (1-\alpha)[(1/\alpha - 1)\zeta\kappa]^{\frac{\xi}{1-\xi}}\right)^{-1/\xi - 1} \\ &\quad \times (1-\alpha)[(1/\alpha - 1)\zeta\kappa]^{\frac{\xi}{1-\xi}}\left(\alpha + (1-\alpha)[(1/\alpha - 1)\zeta\kappa]^{\frac{\xi}{1-\xi}}\right)^{1/\xi} \\ &= -\frac{1}{1-\xi}\left(\frac{(1-\alpha)[(1/\alpha - 1)\zeta\kappa]^{\frac{\xi}{1-\xi}}}{\alpha + (1-\alpha)[(1/\alpha - 1)\zeta\kappa]^{\frac{\xi}{1-\xi}}}\right) \end{aligned} \tag{C.1}$$

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which simplifies as claimed. □

Proof of Lemma 3.4. Demand is a multiple of $\psi'(p)\psi(p)^{-\frac{1}{1-\rho}}$, and so we compute

$$\frac{d}{d\kappa}\left[\psi'(p)\psi(p)^{-\frac{1}{1-\rho}}\right] = \psi''(p)\psi(p)^{-\frac{1}{1-\rho}} - \frac{1}{1-\rho}[\psi'(p)]^2\psi(p)^{-\frac{1}{1-\rho}-1}.$$

Dividing by $\psi'(p)\psi(p)^{-\frac{1}{1-\rho}}p^{-1}$ and using Lemma C.1, we have

$$\epsilon(p;q) = \epsilon(p;\psi') - \frac{\epsilon(p;\psi)}{1-\rho} = \frac{1}{1-\xi}(\epsilon(p;\psi)-1) - \frac{\epsilon(p;\psi)}{1-\rho}$$

which gives the desired expression. To see that $\epsilon(p;q) \geq -(1-\rho)^{-1}$ note that

$$\epsilon(p;q) + \frac{1}{1-\rho} = \frac{(\rho-\xi)}{(1-\xi)(1-\rho)}(1-\epsilon(p;\psi))$$

which is bounded from below by zero by Lemma 3.2 because $\xi \leq \rho$. □

Proof of Proposition 3.5. First note that the revenue of the firm as a function of its price is bounded above by a multiple of $\epsilon(p;\psi)\psi(p)^{-\frac{\rho}{1-\rho}}$ and so its objective vanishes both when the firm equates its price with the marginal cost ($p = \kappa$) and as its price diverges ($p \to \infty$), because when $\xi \leq 0$, $\lim_{p\to\infty}\psi(p)^{-\frac{\rho}{1-\rho}} = 0$, and when $\xi > 0$, $\lim_{p\to\infty}\epsilon(p;\psi) = 0$. The optimal price therefore satisfies the firm's first-order condition.

We now show that there exists a unique solution to this first-order condition. Using Lemma 3.4, the first-order condition (3.16) of the firm becomes

$$\frac{p}{p-\kappa} = -\epsilon(p;q) = \frac{1}{1-\xi}\left(1 + \frac{(\rho-\xi)}{(1-\rho)}\epsilon(p;\psi)\right)$$

which is equivalent to the equation $\Xi(p;\kappa) = 0$ where

$$\Xi(p;\kappa) = -\frac{(1-\rho)}{(1-\xi)} + \frac{(1-\rho)p}{p-\kappa} - \epsilon(p;\psi)\frac{(\rho-\xi)}{(1-\xi)}. \tag{C.2}$$

Note that the elasticity in Lemma 3.2 satisfies $\lim_{p\to\infty}\epsilon(p;\psi) = 1$ when $\xi < 0$ and $\lim_{p\to\infty}\epsilon(p;\psi) = 0$ when $\xi > 0$. Inspection reveals that in both cases

$$\lim_{p\to\infty}\Xi(p;\kappa) = \frac{1}{1-\xi}\left(-\xi(1-\rho) - (\rho-\xi)\lim_{p\to\infty}\epsilon(p;\psi)\right) < 0.$$

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The function $\Xi(p;\kappa)$ therefore has at least one root on (κ,∞) because it diverges as p approaches κ from above. If $\xi<0$, then the right-hand side of (C.3) is everywhere decreasing and so a unique solution to the equation $\Xi(p;\kappa)=0$ is assured. For $\xi>0$, the situation is more complicated. First note that Ξ and its derivative may be written

$$\begin{aligned}\Xi(p;\kappa)&=\frac{-\xi(1-\rho)}{(1-\xi)}+\frac{(1-\rho)\kappa}{p-\kappa}-\epsilon(p;\psi)\frac{(\rho-\xi)}{(1-\xi)}\\ \Xi'(p;\kappa)&=-\frac{(1-\rho)\kappa}{(p-\kappa)^2}-\epsilon'(p;\psi)\frac{(\rho-\xi)}{(1-\xi)}.\end{aligned}\tag{C.3}$$

Recalling that $\epsilon(\kappa;\psi)=\left(1+(1/\alpha-1)^{\frac{1}{1-\xi}}[\kappa\zeta]^{\frac{\xi}{1-\xi}}\right)^{-1}$, we have

$$\begin{aligned}\epsilon'(\kappa;\psi)&=-\frac{(1/\alpha-1)^{\frac{1}{1-\xi}}[\kappa\zeta]^{\frac{\xi}{1-\xi}}\kappa^{-1}}{(1/\xi-1)\left(1+(1/\alpha-1)^{\frac{1}{1-\xi}}[\kappa\zeta]^{\frac{\xi}{1-\xi}}\right)^2}\\ &=-(1/\alpha-1)^{\frac{1}{1-\xi}}[\kappa\zeta]^{\frac{\xi}{1-\xi}}\frac{\kappa^{-1}\epsilon(\kappa;\psi)^2}{1/\xi-1}\end{aligned}$$

and hence

$$\frac{\kappa\epsilon'(\kappa;\psi)}{\epsilon(\kappa;\psi)}=-(1/\alpha-1)^{\frac{1}{1-\xi}}[\kappa\zeta]^{\frac{\xi}{1-\xi}}\frac{\epsilon(\kappa;\psi)}{1/\xi-1}=\frac{\xi(\epsilon(\kappa;\psi)-1)}{1-\xi}$$

and hence $p\epsilon'(p;\psi)=(\epsilon(p;\psi)-1)\epsilon(p;\psi)/(1/\xi-1)$. Note that $\Xi'(p;\kappa)<0$ if and only if $p\Xi'(p;\kappa)<0$, which occurs if and only if

$$\xi(1-\epsilon(p;\psi))\epsilon(p;\psi)\frac{(\rho-\xi)}{(1-\rho)(1-\xi)^2}<\frac{\kappa/p}{(1-\kappa/p)^2}.\tag{C.4}$$

To establish a unique solution to the equation $\Xi(p;\kappa)=0$, we only require $\Xi'(p;\kappa)<0$ for all p satisfying $\Xi(p;\kappa)\geq 0$. Notice that by dividing the expression in (C.3) by $1-\rho$, we see that $\Xi>0$ is equivalent to

$$\frac{1}{p/\kappa-1}>\frac{\epsilon(p;\psi)(\rho-\xi)}{(1-\rho)(1-\xi)}+\frac{\xi}{1-\xi}.\tag{C.5}$$

We therefore want to establish that the inequality (C.4), which is

$$\xi(1-\epsilon(p;\psi))\epsilon(p;\psi)\frac{(\rho-\xi)}{(1-\rho)(1-\xi)^2}<\frac{1}{p/\kappa-1}\left(\frac{1}{p/\kappa-1}+1\right)\tag{C.6}$$

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holds for all p satisfying (C.5). It will therefore suffice to show that for all $\epsilon \in (0,1)$,

$$\frac{\xi(1-\epsilon)\epsilon(\rho-\xi)}{(1-\rho)(1-\xi)^2} < \left(\frac{\epsilon(\rho-\xi)}{(1-\rho)(1-\xi)} + \frac{\xi}{1-\xi}\right)\left(\frac{\epsilon(\rho-\xi)}{(1-\rho)(1-\xi)} + \frac{1}{1-\xi}\right).$$

Multiplying by $(1-\rho)^2(1-\xi)^2$ gives

$$\begin{aligned}\xi(1-\epsilon)\epsilon(\rho-\xi)(1-\rho) &< (\epsilon(\rho-\xi)+\xi(1-\rho))(\epsilon(\rho-\xi)+1-\rho)\\ &= \epsilon^2(\rho-\xi)^2 + \epsilon(\rho-\xi)(1-\rho)(1+\xi) + \xi(1-\rho)^2.\end{aligned}$$

Expanding and rearranging then shows that we require

$$\begin{aligned}0 < Q(\epsilon) &:= [\xi(\rho-\xi)(1-\rho) + (\rho-\xi)^2]\epsilon^2\\ &\quad + \epsilon[(\rho-\xi)(1-\rho)(1+\xi) - \xi(\rho-\xi)(1-\rho)] + \xi(1-\rho)^2\\ &= \rho(1-\xi)(\rho-\xi)\epsilon^2 + \epsilon(\rho-\xi)(1-\rho) + \xi(1-\rho)^2\end{aligned}$$

which holds for all $\epsilon \in (0,1)$ when $\xi \geq 0$. It follows that there exists a unique solution to the firm's first-order condition and that this solves the firm's problem.

To see that $p(\kappa)/\kappa \geq 1/\rho$, note that because $\xi \leq \rho$ and $\epsilon(p;\psi) \leq 1$ for all $p \geq 0$, any root of (C.3) weakly exceeds the solution to $1+(\rho-\xi)/(1-\rho) = (1-\xi)p/(p-\kappa)$, which is $p = \kappa/\rho$. Finally, the comparative statics with respect to κ follow directly from the first-order condition (3.16) together with the fact that $\epsilon(p;q)$ is decreasing in κ for $\xi \in [-\infty, 0)$, increasing in κ for $\xi \in (0,\rho)$ and constant when $\xi = 0$. □

Proof of Lemma 3.7. Using $p(\kappa) = m(\kappa)\kappa$, the holistic markup ratio is

$$\varphi(\kappa) = \frac{\psi(m(\kappa)\kappa)}{\psi(\kappa)} = m(\kappa)\frac{\left(\alpha + (1-\alpha)[(1/\alpha-1)\zeta m(\kappa)\kappa]^{\frac{\xi}{1-\xi}}\right)^{1-1/\xi}}{\left(\alpha + (1-\alpha)[(1/\alpha-1)\zeta\kappa]^{\frac{\xi}{1-\xi}}\right)^{1-1/\xi}}. \tag{C.7}$$

When $\xi = -\infty$ direct calculation shows that $\varphi(\kappa) \equiv 1/\rho < m$, and when $\xi = 0$, we have $\varphi(\kappa) \equiv m(\kappa)^\alpha \leq m(\kappa)$ because $\alpha \in [0,1]$. When $\xi < 0$, note that $m > 1$ implies $m\kappa > \kappa$ and $(m\kappa)^{\frac{\xi}{1-\xi}} < \kappa^{\frac{\xi}{1-\xi}}$ since $\xi/(1-\xi) < 0$. We therefore have $\varphi(\kappa) < m(\kappa)$ everywhere. When $0 < \xi \leq \rho$, the inequality $m\kappa > \kappa$ implies

$$\alpha + (1-\alpha)[(1/\alpha-1)\zeta m\kappa]^{\frac{\xi}{1-\xi}} > \alpha + (1-\alpha)[(1/\alpha-1)\zeta\kappa]^{\frac{\xi}{1-\xi}}$$

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and hence

$$\left(\alpha + (1-\alpha)[(1/\alpha - 1)\zeta m\kappa]^{\frac{\xi}{1-\xi}}\right)^{1-1/\xi} < \left(\alpha + (1-\alpha)[(1/\alpha - 1)\zeta\kappa]^{\frac{\xi}{1-\xi}}\right)^{1-1/\xi}$$

since $1 - 1/\xi < 0$, which completes the proof. □

Proof of Proposition 3.8. The cutoff value of marginal cost $\overline{\kappa}$ is characterized by the indifference relation $\pi(\overline{\kappa}) = f$ for the marginal firm, and aggregate profits vanish if and only if $\int_{\underline{\kappa}}^{\overline{\kappa}} \pi(\kappa)G(d\kappa) = f_e + fG(\overline{\kappa})$. Combining these two conditions and using the fact that $\widehat{\pi}(\kappa)/\pi(\kappa)$ is independent of κ, we obtain equation (3.19). The existence of a unique solution to this last equation then follows by applying Lemma A.4 with $h(\kappa) := \widehat{\pi}(\kappa)$.

To determine the equilibrium mass of firms in the environment with firm heterogeneity, first recall that the demand for a consumption experience satisfies

$$c(p) = (\rho/\lambda)^{\frac{1}{1-\rho}} \psi(p)^{-\frac{1}{1-\rho}}$$

where λ is the Lagrange multiplier on the budget constraint. Since aggregate ex-ante profits are zero, the budget constraint becomes $\int_{\underline{\kappa}}^{\overline{\kappa}} \psi(p(\kappa))c(\kappa)M_e G(d\kappa) = \overline{T}$ and substitution of the above implies that the multiplier λ solves

$$(\rho/\lambda)^{\frac{1}{1-\rho}} = \frac{\overline{T}/M_e}{\int_{\underline{\kappa}}^{\overline{\kappa}} \psi(p(\kappa))^{-\frac{1}{1-\rho}} G(d\kappa)}.$$

Using $q(p(\kappa)) = \psi'(p(\kappa))c(p(\kappa))$, the profits of a firm of type κ are then

$$\pi(\kappa) = \frac{(\overline{T}/M_e)(p(\kappa) - \kappa)}{\int_{\underline{\kappa}}^{\overline{\kappa}} \psi(p(\kappa))^{-\frac{\rho}{1-\rho}} G(d\kappa)} \psi'(p(\kappa))\psi(p(\kappa))^{-\frac{1}{1-\rho}}.$$

It follows that $\pi(\overline{\kappa}) = f$ if and only if the mass of firms satisfies (3.20). When firms are homogeneous, aggregate profits must equate with the sum of both fixed and operating costs, or

$$\pi(\overline{\kappa}) = (p(\overline{\kappa}) - \overline{\kappa})q(\overline{\kappa}) = f + f_e. \tag{C.8}$$

Rearranging the budget constraint in the consumer problem (3.13) and using the fact that $\overline{c} = \overline{q}/\psi'(\overline{p})$, we have $\psi(\overline{p})(\overline{q}/\psi'(\overline{p}))M_e = \psi(\overline{p})\overline{c}M_e = \overline{T}$ and hence $\overline{pq}M_e = \epsilon(\overline{p};\psi)\overline{T}$. Combining this last equality with (C.8) then gives (3.22). □

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D Welfare analysis

This appendix provides of proofs of all the claims in the main text comparing the efficient and equilibrium allocations. Recall from (4.1) that for any continuous, decreasing positive-valued function F on $[\underline{\kappa}, \infty)$ we define

$$J(\overline{\kappa}; F) := \int_{\underline{\kappa}}^{\overline{\kappa}} (F(\kappa)/F(\overline{\kappa}))G(d\kappa).$$

Lemma D.1, Lemma D.2, Lemma D.3 and Lemma D.4 are all technical observations that will be used in the proof of Proposition 4.1.

Lemma D.1. *If $F, H : [\underline{\kappa}, \infty) \to \mathbb{R}$ are two continuously differentiable, strictly decreasing, positive-valued functions satisfying $F'(x)/F(x) \geq H'(x)/H(x)$ for all $x \in [\underline{\kappa}, \infty)$, then $J(\kappa; F) \leq J(\kappa; H)$ for all $\kappa \in [\underline{\kappa}, \infty)$.*

Proof. First note that $J(\underline{\kappa}; F) = J(\underline{\kappa}; H) = 0$ and that the derivative of J is

$$\begin{aligned} J'(\overline{\kappa}; F) &= g(\overline{\kappa}) - F'(\overline{\kappa}) \int_{\underline{\kappa}}^{\overline{\kappa}} (F(\kappa)/F(\overline{\kappa})^2)G(d\kappa) \\ &= g(\overline{\kappa}) + [-F'(\overline{\kappa})/F(\overline{\kappa})]J(\overline{\kappa}; F). \end{aligned}$$

We now define $h(\overline{\kappa}) := J(\overline{\kappa}; H) - J(\overline{\kappa}; F)$ and use the above to write

$$\begin{aligned} h'(\overline{\kappa}) &= J'(\overline{\kappa}; H) - J'(\overline{\kappa}; F) \\ &= [-H'(\overline{\kappa})/H(\overline{\kappa})]J(\overline{\kappa}; H) - [-F'(\overline{\kappa})/F(\overline{\kappa})]J(\overline{\kappa}; F) \\ &= ([-H'(\overline{\kappa})/H(\overline{\kappa})] - [-F'(\overline{\kappa})/F(\overline{\kappa})])J(\overline{\kappa}; F) \\ &+ [-H'(\overline{\kappa})/H(\overline{\kappa})](J(\overline{\kappa}; H) - J(\overline{\kappa}; F)) \\ &= j(\overline{\kappa})J(\overline{\kappa}; F) + [-H'(\overline{\kappa})/H(\overline{\kappa})]h(\overline{\kappa}) \end{aligned}$$

where we defined $j(\overline{\kappa}) := [-H'(\overline{\kappa})/H(\overline{\kappa})] - [-F'(\overline{\kappa})/F(\overline{\kappa})] \geq 0$. It follows that $h(\underline{\kappa}) = 0$ and $h'(\overline{\kappa}) \geq [-H'(\overline{\kappa})/H(\overline{\kappa})]h(\overline{\kappa})$ for all $\overline{\kappa} \geq \underline{\kappa}$, and so the desired non-negativity of h follows from an application of Gronwall's inequality (see, e.g., Ziebur (1968)) to the function $u(\overline{\kappa}) \equiv -h(\overline{\kappa})$. □

Lemma D.2 builds upon Lemma D.1 to show that determining whether the marginal cost of the marginal firm is too high or too low in equilibrium amounts to comparing the markup with the elasticity of the holistic marginal cost.

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Lemma D.2. *The equilibrium $\overline{\kappa}$ will be inefficiently high if*

$$p(\kappa)/\kappa - 1 \geq \frac{1/\rho - 1}{\epsilon(\kappa;\psi)} \tag{D.1}$$

for all κ, and inefficiently low when the reverse inequality holds for all κ.

Proof. Recall from Section 4.1 that the equilibrium $\overline{\kappa}$ will be inefficiently high if $J(\kappa;\widehat{\pi}) \leq J(\kappa;\psi^{-\frac{\rho}{1-\rho}})$ for all κ and inefficiently low if the reverse inequality holds for all κ. Combining this observation with Lemma D.1 we see that the equilibrium $\overline{\kappa}$ will be inefficiently high if

$$\epsilon(\kappa;\widehat{\pi}) \geq \epsilon(\kappa;\psi^{-\frac{\rho}{1-\rho}}) = -\frac{\epsilon(\kappa;\psi)}{1/\rho - 1} \tag{D.2}$$

and inefficiently low if the reverse inequality holds for all κ. Now note that by the envelope theorem, we have $\widehat{\pi}'(\kappa) = -\psi'(p(\kappa))\psi(p(\kappa))^{-\frac{1}{1-\rho}}$, and hence

$$\epsilon(\kappa;\widehat{\pi}) = \frac{\kappa\widehat{\pi}'(\kappa)}{\widehat{\pi}(\kappa)} = \frac{-\kappa\psi'(p(\kappa))\psi(p(\kappa))^{-\frac{1}{1-\rho}}}{(p(\kappa)-\kappa)\psi'(p(\kappa))\psi(p(\kappa))^{-\frac{1}{1-\rho}}} = -\frac{1}{p(\kappa)/\kappa - 1}. \tag{D.3}$$

Substitution then implies that inequality (D.2) is equivalent to

$$-\frac{1}{p(\kappa)/\kappa - 1} \geq -\frac{\epsilon(\kappa;\psi)}{1/\rho - 1}$$

which gives the desired conclusion upon rearrangement. □

Lemma D.2 shows that establishing Proposition 4.1 amounts to comparing the elasticity of the holistic marginal cost with the equilibrium markup. Before turning to the proof we record two more observations.

Lemma D.3. *If $\xi < 0$, then the inequality*

$$(1 - \rho/\xi + z)(1+z)^{\xi/(1-\xi)} > \rho^{\frac{1}{1-\xi}}(1 - 1/\xi). \tag{D.4}$$

holds for all $\rho \in (0,1)$ and $z \geq 0$.

Proof. We will prove the inequality (D.4) by showing that it holds for $z = 0$ and that the left-hand side is increasing in z. For $z = 0$, (D.4) becomes $1 - \rho/\xi > \rho^{\frac{1}{1-\xi}}(1 - 1/\xi)$, which for $\xi < 0$ is equivalent to $\xi - \rho < \rho^{\frac{1}{1-\xi}}(\xi - 1)$ or $\xi < \rho - \rho^{\frac{1}{1-\xi}} + \rho^{\frac{1}{1-\xi}}\xi$. This

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last inequality is an equality at $\rho = 1$ and the derivative with respect to ρ of the right-hand side is $1 - \rho^{\frac{\xi}{1-\xi}}/(1-\xi) + \rho^{\frac{\xi}{1-\xi}}\xi/(1-\xi) = 1 - \rho^{\frac{\xi}{1-\xi}} < 0$, which shows that equation (D.4) holds for $z = 0$. We now take logarithms of the left-hand side of (D.4) and take the derivative with respect to z to obtain

$$\frac{1}{1-\rho/\xi+z} + \frac{(1+z)^{-1}}{1/\xi - 1}. \tag{D.5}$$

The expression in (D.5) is positive for $z \geq 0$ if and only if $1 - 1/\xi + z(1-1/\xi) > 1 - \rho/\xi + z$, or $-z/\xi > (1-\rho)/\xi$, which is always true. □

Lemma D.4. *If $\xi \in (0, \rho)$ then the inequality*

$$(1+z)^{\xi/(1-\xi)}[\rho - \xi(1+z)] < \rho^{1/(1-\xi)}(1-\xi) \tag{D.6}$$

holds for all $z \geq 0$ and $\rho \in (0,1)$.

Proof. When $z = 0$, inequality (D.6) becomes $0 < \rho^{1/(1-\xi)}(1-\xi) + \xi - \rho$. For $\rho = 1$ this last inequality becomes an equality, and the derivative of the right-hand side with respect to ρ is $\rho^{\xi/(1-\xi)} - 1$, which is negative for all $\rho \in (0,1)$. This ensures that the inequality (D.6) holds for all $\rho \in (0,1)$ when $z = 0$. To establish (D.6) for all $z \geq 0$, note that we may divide both sides by ρ and rearrange to obtain

$$0 < \rho^{\xi/(1-\xi)}(1-\xi) + (1+z)^{1/(1-\xi)}(\xi/\rho) - (1+z)^{\xi/(1-\xi)}.$$

The derivative of the right-hand side of this last inequality with respect to z is

$$(1+z)^{1/(1-\xi)-1}(\xi/\rho)/(1-\xi) - (1+z)^{\xi/(1-\xi)-1}\xi/(1-\xi),$$

which is non-negative if and only if $\xi \leq (1+z)(\xi/\rho)$, which is true for all $z \geq 0$. □

Proof of Proposition 4.1. The claims regarding $\xi \in \{-\infty, 0\}$ follow from the explicit calculations stated in Lemma A.3 and Lemma A.5, because in these cases the ratio $\psi(\kappa)^{-\frac{\rho}{1-\rho}}/\widehat{\pi}(\kappa)$ is constant in κ and so the equations (3.8) and (3.19) governing the efficient and equilibrium cutoff values for $\overline{\kappa}$ coincide.

For the remaining claims, we define

$$\widehat{p}(\kappa)/\kappa := \frac{1/\rho - 1}{\epsilon(\kappa;\psi)} + 1 \tag{D.7}$$

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and note that by Lemma D.2 it will suffice to show that for $\xi \in [-\infty, 0)$ we have

$$p(\kappa)/\kappa - 1 \geq \widehat{p}(\kappa)/\kappa - 1 \tag{D.8}$$

for all $\kappa \geq \underline{\kappa}$, and that for $\xi \in (0, \rho]$ we have the reverse inequality to (D.8). As shown in the proof of Proposition 3.5, the price $p(\kappa)$ is the unique solution to the equation $\Xi(p;\kappa) = 0$, where Ξ is defined in equation (C.3) and satisfies $\lim_{p\to\kappa^+} \Xi(p;\kappa) = \infty$ and $\lim_{p\to\infty} \Xi(p;\kappa) < 0$. To establish (D.8) we then want to show that $\Xi(\widehat{p}(\kappa);\kappa) > 0$ for all $\kappa > 0$ when $\xi < 0$, and $\Xi(\widehat{p}(\kappa);\kappa) < 0$ for all $\kappa > 0$ when $\xi \in (0, \rho)$.

For ease of notation, for any $\kappa \geq \underline{\kappa}$ we define

$$z(\kappa) := (1-\rho)(1/\alpha - 1)^{\frac{1}{1-\xi}} [\zeta\kappa]^{\frac{\xi}{1-\xi}}. \tag{D.9}$$

Note that by Lemma 3.2, $z(\kappa)$ and $\widehat{p}(\kappa)$ satisfy

$$\rho\widehat{p}(\kappa)/\kappa = (1-\rho)\Big(1 + (1/\alpha - 1)^{\frac{1}{1-\xi}} [\kappa\zeta]^{\frac{\xi}{1-\xi}}\Big) + \rho = 1 + z(\kappa).$$

Substitution gives

$$\Xi(\widehat{p}(\kappa);\kappa) = -\frac{(1-\rho)\xi}{(1-\xi)} + \rho\epsilon(\kappa;\psi) - \epsilon(\widehat{p}(\kappa);\psi)\frac{(\rho-\xi)}{(1-\xi)}. \tag{D.10}$$

We first suppose that $\xi < 0$, in which case $\Xi(\widehat{p}(\kappa);\kappa) > 0$ is equivalent to

$$(\rho-\xi)\epsilon(\widehat{p}(\kappa);\psi) < \rho(1-\xi)\epsilon(\kappa;\psi) - (1-\rho)\xi. \tag{D.11}$$

When $\xi < 0$, both sides of (D.11) are positive and so taking the reciprocal and simplifying, we see that the desired inequality is equivalent to

$$\begin{aligned} \epsilon(\widehat{p}(\kappa);\psi)^{-1} - 1 &> \frac{\rho(1-\xi)[1-\epsilon(\kappa;\psi)]}{\rho(1-\xi)\epsilon(\kappa;\psi) - (1-\rho)\xi} \\ &= \frac{\rho(1-\xi)[\epsilon(\kappa;\psi)^{-1} - 1]}{\rho - \xi - (1-\rho)\xi[\epsilon(\kappa;\psi)^{-1} - 1]}. \end{aligned} \tag{D.12}$$

Using Lemma 3.2, we have $\epsilon(\kappa;\psi)^{-1} - 1 = (1/\alpha - 1)^{\frac{1}{1-\xi}} [\kappa\zeta]^{\frac{\xi}{1-\xi}}$ and hence

$$\frac{\epsilon(\widehat{p}(\kappa);\psi)^{-1} - 1}{\epsilon(\kappa;\psi)^{-1} - 1} = [\widehat{p}(\kappa)/\kappa]^{\frac{\xi}{1-\xi}}. \tag{D.13}$$

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Multiplying (D.12) by $\rho^{\frac{\xi}{1-\xi}}$ and dividing by $\epsilon(\kappa;\psi)^{-1}-1$ and then using (D.13), the desired inequality is equivalent to

$$(1+z(\kappa))^{\xi/(1-\xi)} = [\rho\widehat{p}(\kappa)/\kappa]^{\xi/(1-\xi)} > \frac{\rho^{\frac{1}{1-\xi}}(1-\xi)}{\rho-\xi-(1-\rho)\xi(1/\alpha-1)^{\frac{1}{1-\xi}}[\zeta\kappa]^{\frac{\xi}{1-\xi}}} = \frac{\rho^{\frac{1}{1-\xi}}(1-1/\xi)}{1-\rho/\xi+z(\kappa)},$$

which is true by Lemma D.3. This deals with the $\xi<0$ case.

For $\xi>0$, the desired inequality $\Xi(\widehat{p}(\kappa);\kappa)<0$ is equivalent to

$$(1-\rho)\xi+(\rho-\xi)\epsilon(\widehat{p}(\kappa);\psi) > \rho(1-\xi)\epsilon(\kappa;\psi). \tag{D.14}$$

Since $\xi\in(0,\rho)$, both sides of (D.14) are positive and so taking the reciprocal, we see that the desired inequality is equivalent to

$$\frac{\rho(1-\xi)}{(1-\rho)\xi+(\rho-\xi)\epsilon(\widehat{p}(\kappa);\psi)} < \frac{1}{\epsilon(\kappa;\psi)}.$$

This is equivalent to

$$\frac{\rho(1-\xi)\epsilon(\widehat{p}(\kappa);\psi)}{(1-\rho)\xi+(\rho-\xi)\epsilon(\widehat{p}(\kappa);\psi)} < \frac{\epsilon(\widehat{p}(\kappa);\psi)}{\epsilon(\kappa;\psi)}$$
$$\frac{\epsilon(\kappa;\psi)}{\epsilon(\widehat{p}(\kappa);\psi)} < \frac{(1-\rho)\xi+(\rho-\xi)\epsilon(\widehat{p}(\kappa);\psi)}{\rho(1-\xi)\epsilon(\widehat{p}(\kappa);\psi)}$$

Now, subtracting 1 from both sides and using equation (D.13) we have

$$\begin{aligned}\frac{\epsilon(\kappa;\psi)}{\epsilon(\widehat{p}(\kappa);\psi)}-1 &< \frac{(1-\rho)\xi}{\rho(1-\xi)}\frac{(1-\epsilon(\widehat{p}(\kappa);\psi))}{\epsilon(\widehat{p}(\kappa);\psi)}\\ &= \frac{(1-\rho)\xi}{\rho(1-\xi)}[\widehat{p}(\kappa)/\kappa]^{\frac{\xi}{1-\xi}}(\epsilon(\kappa;\psi)^{-1}-1)\\ &= \frac{(1-\rho)\xi}{\rho(1-\xi)}[\widehat{p}(\kappa)/\kappa]^{\frac{\xi}{1-\xi}}(1/\alpha-1)^{\frac{1}{1-\xi}}[\kappa\zeta]^{\frac{\xi}{1-\xi}}.\end{aligned}$$

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The desired inequality is then

$$\frac{\rho(1-\xi)\epsilon(\widehat{p}(\kappa);\psi)}{(1-\rho)\xi+(\rho-\xi)\epsilon(\widehat{p}(\kappa);\psi)} < \frac{\epsilon(\widehat{p}(\kappa);\psi)}{\epsilon(\kappa;\psi)}$$
$$\frac{\epsilon(\kappa;\psi)}{\epsilon(\widehat{p}(\kappa);\psi)} - 1 < \frac{(1-\rho)\xi}{\rho(1-\xi)\epsilon(\widehat{p}(\kappa);\psi)} - \frac{\xi(1-\rho)}{\rho(1-\xi)}$$

which simplifies to

$$[\widehat{p}(\kappa)/\kappa]^{\xi/(1-\xi)} - 1 < \frac{\xi(1-\rho)}{\rho(1-\xi)}[\widehat{p}(\kappa)/\kappa]^{\xi/(1-\xi)}\epsilon(\kappa;\psi)^{-1}.$$

Further rearrangement gives

$$[\widehat{p}(\kappa)/\kappa]^{\xi/(1-\xi)}\left[1 - \frac{\xi(1-\rho)}{\rho(1-\xi)}\epsilon(\kappa;\psi)^{-1}\right] < 1$$
$$[\rho\widehat{p}(\kappa)/\kappa]^{\xi/(1-\xi)}\left[1-\xi-(\xi/\rho)(1-\rho)\epsilon(\kappa;\psi)^{-1}\right] < \rho^{\xi/(1-\xi)}(1-\xi).$$

Using (D.9) we have

$$\begin{aligned}\rho(1-\xi)-\xi(1-\rho)\epsilon(\kappa;\psi)^{-1} &= \rho(1-\xi)-\xi(1-\rho)(1+(1/\alpha-1)^{\frac{1}{1-\xi}}[\zeta\kappa]^{\frac{\xi}{1-\xi}}) \\ &= \rho-\xi-\xi z(\kappa).\end{aligned}$$

which is equivalent to $(1+z(\kappa))^{\xi/(1-\xi)}[\rho-\xi-\xi z(\kappa)] < \rho^{1/(1-\xi)}(1-\xi)$, and is therefore true by Lemma D.4. □

Proof of Proposition 4.2. First note that by the definition of $\widehat{\pi}$, for all κ we have

$$\begin{aligned}\frac{\widehat{\pi}(\kappa)}{\psi(p(\kappa))^{-\frac{\rho}{1-\rho}}} &= \frac{(p(\kappa)-\kappa)\psi'(p(\kappa))\psi(p(\kappa))^{-\frac{1}{1-\rho}}}{\psi(p(\kappa))^{-\frac{\rho}{1-\rho}}} \\ &= (1-\kappa/p(\kappa))\epsilon(p(\kappa);\psi).\end{aligned} \tag{D.15}$$

Using equation (D.15) and Proposition 3.8, the total resources devoted to setting up

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firms in the monopolistically competitive equilibrium is given by

$$M_e(f_e + fG(\overline{\kappa})) = \frac{\overline{T}\int_{\underline{\kappa}}^{\overline{\kappa}} \widehat{\pi}(\kappa)G(d\kappa)}{\int_{\underline{\kappa}}^{\overline{\kappa}} \psi(p(\kappa))^{-\frac{\rho}{1-\rho}} G(d\kappa)} = \frac{\overline{T}\int_{\underline{\kappa}}^{\overline{\kappa}} (1-\kappa/p(\kappa))\epsilon(p(\kappa);\psi)\psi(p(\kappa))^{-\frac{\rho}{1-\rho}} G(d\kappa)}{\int_{\underline{\kappa}}^{\overline{\kappa}} \psi(p(\kappa))^{-\frac{\rho}{1-\rho}} G(d\kappa)}$$

while by Proposition 3.3 in the efficient allocation it is simply $(1-\rho)\overline{T}$. It will therefore suffice to show that $(1-\kappa/p(\kappa))\epsilon(p(\kappa);\psi) \leq 1-\rho$ for all κ, with equality if and only if $\xi = -\infty$. To this end, note that from the proof of Proposition 3.5 that $p(\kappa)$ solves

$$\Xi(p;\kappa) = \frac{-\xi(1-\rho)}{(1-\xi)} + \frac{(1-\rho)\kappa}{p-\kappa} - \epsilon(p;\psi)\frac{(\rho-\xi)}{(1-\xi)} = 0. \tag{D.16}$$

It follows that

$$(1-\kappa/p(\kappa))\epsilon(p(\kappa);\psi)(\rho-\xi) = -\xi(1-\rho)(1-\kappa/p(\kappa)) + (1-\xi)(1-\rho)\kappa/p(\kappa)$$

which simplifies to

$$(1-\kappa/p(\kappa))\epsilon(p(\kappa);\psi) = (1-\rho)\left(\frac{\kappa/p(\kappa)-\xi}{\rho-\xi}\right).$$

The equilibrium mass of firms will therefore be inefficiently low if $\kappa/p(\kappa) - \xi < \rho - \xi$, which is always true for $\xi \neq -\infty, \rho$ by Proposition 3.5. When $\xi = -\infty$, direct calculation shows that $(1-\kappa/p(\kappa))\epsilon(p(\kappa);\psi) = 1-\rho$, while for $\xi = \rho$, we have $(1-\kappa/p(\kappa))\epsilon(p(\kappa);\psi) = (1-\rho)\epsilon(p(\kappa);\psi) < 1-\rho$. □

Proof of Proposition 4.3. Proposition 4.1 shows that firm selection is efficient if and only if $\xi = -\infty$ or $\xi = 0$, and so the equilibrium allocation is certainly not efficient outside of these two cases. Further, Proposition 4.2 shows that $M_e(f_e + fG(\overline{\kappa}))$ is inefficiently low when $\xi = 0$, and so the equilibrium in this case also cannot be Pareto efficient.

It remains to deal with the $\xi = -\infty$ case. Proposition 4.1 and Proposition 4.2 combine to show that in this case, the value of M_e coincides with the value chosen by the planner. Since $\psi'(p(\kappa)) = \psi'(\kappa) = 1$ in this case, it remains to establish that the efficient quantities in (3.10) coincide with the equilibrium consumption quantities in

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(3.21). Since the cutoff $\overline{\kappa}$ and mass M_e coincide, this requires

$$\frac{\psi(p(\kappa))^{-\frac{1}{1-\rho}}}{\int_{\underline{\kappa}}^{\overline{\kappa}} \psi(p(\kappa))^{-\frac{\rho}{1-\rho}} G(d\kappa)} = \frac{\rho\psi(\kappa)^{-\frac{1}{1-\rho}}}{\int_{\underline{\kappa}}^{\overline{\kappa}} \psi(\kappa)^{-\frac{\rho}{1-\rho}} G(d\kappa)}. \tag{D.17}$$

Using the fact that $\psi(p(\kappa)) = \psi(\kappa)/\rho$ for all κ when $\xi = -\infty$, the left-hand side of equation (D.17) becomes

$$\frac{\psi(p(\kappa))^{-\frac{1}{1-\rho}}}{\int_{\underline{\kappa}}^{\overline{\kappa}} \psi(p(\kappa))^{-\frac{\rho}{1-\rho}} G(d\kappa)} = \frac{\rho^{\frac{1}{1-\rho}}\psi(\kappa)^{-\frac{1}{1-\rho}}}{\rho^{\frac{\rho}{1-\rho}} \int_{\underline{\kappa}}^{\overline{\kappa}} \psi(\kappa)^{-\frac{\rho}{1-\rho}} G(d\kappa)}$$

which is exactly the right-hand side of equation (D.17). □

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