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ABSTRACT

Accelerating demand for wireless broadband is accentuating the need to optimize use of limited spectrum resources that are susceptible to congestion. Recent technological innovations enable exclusive-use, licensed spectrum and open-access, unlicensed spectrum to serve as complementary goods. We present a game-theoretic model in which wireless broadband service providers engage in simultaneous pricing and service decisions for a heterogeneous consumer population. We demonstrate that for some unlicensed allocations, service providers may maximize profit by offloading some consumer traffic onto the unlicensed band. Consequently, adding unlicensed capacity can increase congestion in wireless spectrum bands in ways that harm total and consumer welfare. These effects are reminiscent of Braess's Paradox, in which adding capacity counterintuitively leads to greater congestion. Notably, these effects emerge through supply-side differentiation strategies, rather than demand-side responses. We then utilize our framework to analyze recent high-profile decisions by the FCC and introduce a framework for identifying the appropriate balance between licensed and unlicensed allocations.

1 INTRODUCTION

The emergence of mobile broadband is one of the most dramatic technological and economic developments of the past half-century. Demand for ubiquitous connectivity and rapid innovation have elevated mobile wireless communications from a specialized voice service into a dominant technology that now supports a majority of all Internet protocol traffic (Cisco 2019). Sustaining this growth in mobile services is that all wireless transmissions depends on a scarce, common-pool resource: the electromagnetic spectrum. A coherent regime for allocating spectrum to different uses is necessary to avoid interference conflicts, manage congestion, and induce investment.

Spectrum allocation has evolved into an ongoing attempt to strike a balance between two widely divergent approaches. A series of statutes enacted in the early 20th century mandated that everyone engaging in radio communications first obtain a license grant for the exclusive right to use a portion of the spectrum. These exclusive licenses initially supported the emergence of a vibrant radio and television broadcasting industry and, in recent decades, rapid growth in cellular telephony and— most importantly—wireless broadband. A different design was introduced in 1938, when the Federal Communications Commission ("FCC") began authorizing low power devices operating without a spectrum license. Unlicensed spectrum initially supported fairly modest innovations, such as garage door openers, wireless microphones, cordless telephones, baby monitors, and television remote controls. The creation of Wi-Fi and Bluetooth in the late 1990s ushered in a new era in which unlicensed spectrum technologies emerged as major drivers of economic growth.

The success of both approaches has forced the FCC to decide how much spectrum to assign to licensed and unlicensed uses. The agency has failed, however, to develop a conceptual framework, resulting in a process that is unpredictably ad hoc. In 2020 alone, the FCC initiated three separate proceedings according three distinct designs: the 3.7 GHz proceeding devoted the entire block to exclusive licenses (FCC 2020a), the 5.9 GHz proceeding split the band into roughly equal licensed and unlicensed allocations (FCC 2020b), and the 6 GHz proceeding designated all available spectrum for unlicensed access (FCC 2020c).

The unpredictable nature of these decisions is exacerbated by a policy discourse that views these regimes as mutually exclusive alternatives, with any allocation of unlicensed spectrum harming licensed uses and vice versa. Framing the issue as a zero-sum game overlooks the fact that in practice service providers ("*SP*s") treat licensed and unlicensed spectrum as complements, with *SP*s able to shift different traffic between licensed to unlicensed bands.

We inform this discourse by offering a game-theoretic model that reflects the complementarity between licensed and unlicensed spectrum, while capturing the unusual dynamic in which possessors of exclusive licenses may represent sources of congestion in unlicensed spectrum. We model the strategic decisionmaking, payoffs, and welfare considerations of an agent (e.g., an *SP*) that maximizes profit by utilizing a pair of instruments—price setting and dictating the amount of traffic it facilitates over its exclusive allocation. Our model captures key economic features of spectrum, including its susceptibility to congestion, implicating the literatures on pricing congestible resources and impure public goods. We then demonstrate how these congestion dynamics can give rise to a form of Braess's Paradox, in which adding capacity may counterintuitively worsen congestion in unlicensed bands and reduce overall welfare.

The structure of the Article is as follows. Section 2 provides relevant background and identifies technological and economic concepts presented by the model. Section 3 formally introduces the model and explains its formulation. Section 4 explores the dynamics of the model, particularly how the noncooperative strategies of a licensed service provider give rise to a complex set of shifts in strategy and economic welfare. Section 5 conducts a robustness analysis that shows that the Braess's Paradox effects identified by the model exist for a wide range of input conditions. Section 6 employs the model to offer a welfare assessment of the widely divergent allocation decisions recently taken by the FCC in three high-profile spectrum allocations. Section 7 concludes.

2 BACKGROUND AND LITERATURE REVIEW

Modern spectrum policy began when the Radio Act of 1912 mandated that everyone using radio communications obtain a license from the Secretary of Commerce and Labor. After a court decision held that the Act did not give the Secretary the discretion to deny any license application, Congress enacted the Radio Act of 1927, which gave the newly created Federal Radio Commission the authority to issue exclusive licenses. The Communications Act of 1934 transferred these responsibilities to the FCC, which created an administrative system for allocating spectrum to specific uses and for assigning licenses to specific users (*NBC v. United States*, 319 U.S. 190 (1943)).

Ronald Coase's (1959) landmark article demonstrated that the FCC could forego administrative allocation of spectrum if it created well-defined spectrum rights. As noted above, Coase's (1959) groundbreaking work dispelled the myth that interference spillovers required government intervention, giving rise to the Coase Theorem (1960) and providing a powerful argument for assigning property rights through the price system. Coase's analysis of how market transactions of property rights can address externalities would underly contributions by Harold Demsetz (1967), showing how increases in value can promote shifts from shared property to exclusive property rights At roughly the same time, Garrett Hardin's (1968) article on "The Tragedy of the Commons" explored the free-rider problem that emerges from nonexclusive ownership of property.

These works spawned a body of scholarship favoring exclusive property rights in spectrum[1] and Congressional action authorizing the FCC to use auctions to grant spectrum licenses to the highest bidder.[2] The result has been significant investment in a fixed-cost intensive industry, robust competition between commercial providers in deploying service, and a source of incremental revenue that available to satisfy the statutory pay-as-you-go requirement that any new expenditures be offset either by new revenue or by spending cuts.

The FCC began experimenting with unlicensed spectrum in 1938, when it allowed companies to produce low-power, non-interfering radio devices without obtaining a spectrum license. During the 1970s, members of the technical community began to experiment with other unlicensed uses (Dixon 1976; Newhouse 1978). Most notably, the IEEE began exploring Wi-Fi

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[1] For early examples, see De Vany et al. (1969) and Minasian (1975). For more modern examples, see Rostom and Steinberg (1997); White (2001); Owen and Rosston (2002); Hazlett (2005); and Faulhaber (2003, 2005).

[2] Spectrum assignment via competitive bidding in an auction setting was first authorized in 1993 for commercial wireless communications. Prior to the authorization, spectrum licenses were awarded through regulatory processes (e.g., specific applications and comparative hearings) or lotteries. See Omnibus Budget Reconciliation Act of 1993, Pub. L. No. 103-66, § 6002(a), 107 Stat. 312, 387-92 (1993) (codified at 47 U.S.C.§ 309(j)).

in 1988 and issued its first standard in 1997. A wave of legal scholarship soon emerged advocating for broader use of unlicensed spectrum as a driver for innovation and economic growth (Benkler 1998; Lessig 2001; Werbach 2004; Goodman 2004).

More recently, technologies have emerged that interoperate on both licensed and unlicensed spectrum. Nascent mobile services, such as LTE-Unlicensed ("LTE-U") and the proposed 5G New Radio-Unlicensed ("NR-U"), treat licensed and unlicensed spectrum as complementary resources for augmenting cellular service by shifting traffic from licensed to unlicensed spectrum. In effect, this level of portability adds complexity, opportunities for suboptimal resource provisioning, and path dependence in last-mile service. Notwithstanding the growing prevalence of these resource economics, little study has been done. Nguyen et al. present the work most closely related to ours by modeling access to television white-space bands by homogeneous (2011a) and heterogeneous (2011b) consumers. Because of the cumbersome access requirements imposed on white-space bands, however, these articles theorize a rigid and narrow set of events that is confined to consumer demand-side responses that have failed to materialize (McFadden 2021).[3]

We reframe the traditional conception of treating these property paradigms as mutually exclusive alternatives within the scope of rights in spectrum and instead analyze them as interoperable complements that an *SP* may possess and use when provisioning service. By analyzing these allocations together as a hybrid system within the spectrum of property, new

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[3] Under the television white spaces-driven approach, an *SP* sets prices, and those prices then induce consumers to switch traffic to adjacent resources. This sequence is a consequence of television white space operations, in which specialized user devices register to, and interact with, a spectrum database that incorporates geo-location before accessing inactive UHF/VHF spectrum bands. Our model extends this approach by accommodating both demand-side and supply-side responses to congestion, with a focus on the latter.

incentives and dynamics emerge (Heller 2013). We illuminate these features by offering a networking-driven model that presents strategic, unilateral actions of a possessor of licensed spectrum (e.g., an *SP*) that shifts data traffic between licensed and unlicensed spectrum allocations based on local information. Our model emphasizes supply-side responses, in which *SP*s manage congestion by deciding whether to use licensed or unlicensed spectrum to offer particular services to different types of consumers. This supplier-driven routing flexibility postdates the literature referenced above (ETSI 2016) and, to our knowledge, has never been researched in the context we present. It also introduces previously unaddressed congestion dynamics that can drive welfare outcomes reminiscent of Braess's Paradox, where adding capacity to a network platform may *increase* congestion and *reduce* total surplus. We then apply our model to recent regulatory decisions to inform stakeholders of the tradeoffs inherent in decisions whether to devote spectrum to licensed or unlicensed uses.

Our work intersects with, and contributes to, several strands of academic literature, each of which is addressed below.

2.1 Price Competition and Congestible Resources

Information theory has long recognized that increases in noise reduce the effective carrying capacity of any bandwidth-limited channel (Shannon 1948). Neighboring transmissions represent one of the most important sources of bandwidth-reducing noise and bring spectrum usage squarely within the literature on pricing congestible resources (Gupta and Kumar 2000).

The transportation[4] and operations research literatures[5] have written extensively on price competition for congestible resources. In each discipline, however, articles present competition for a single, excludible resource.

We extend this discussion by introducing a second, *nonexclusive* resource that coexists with an excludable resource allocation. Adding unlicensed spectrum stimulates competition and compels *SP*s to differentiate service between exclusive and nonexclusive resources based on resource congestion. The crux of our analysis is identifying where congestion dominates pricing behavior, effectively compelling an *SP* to differentiate service through exclusivity rather than undergoing a traditional price reduction to increased competition.

2.2 Public Goods

Modeling a nonexclusive, shared resource—such as unlicensed spectrum bands—introduces the economics of public goods. Paul Samuelson (1954, 1964) first demonstrated that even with a right to exclude, nonrivalry limits firms' ability to induce consumers to reveal the intensity of their preferences. This incentive incompatibility is what causes markets to underproduce public goods.

Samuelson (1958) recognized that congestion could make public goods more incentive compatible but thought that it was impossible to determine whether the resulting equilibrium was socially optimal. Congestion was later emphasized in the literature of impure public goods including James Buchanan's (1965) theory of club goods and Charles Tiebout's (1956) work on

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[4] For early examples involving urban traffic congestion, see Buchanan (1952); Walters (1961); Smeed (1968). For modern designs of congestion tolls and variable traffic, see Arnott et al. (2005); Levinson (2005); Zhang et al. (2011); Basso et al. (2021).

[5] *See* MacKie-Mason and Varian (1995); Ha et al. (2003); Acemoglu & Ozdaglar (2007); Morrison & Whinston (2007); De Borger et al. (2008); Perakis and Sun (2014).

"local public goods." These works helped illuminate that congestion presents a new dimension where public goods can vary in *quality*, with these variations enabling consumers to reveal their preferences by reallocating purchases.

Our work contributes to this literature by defining a multi-resource marketplace where private and public goods serve as functional complements for *SP*s. Because both licensed and unlicensed allocations are subject to congestion, variations in quality of service ("QoS") can give rise to de facto markets that are amenable to preference revelation (for an overview, see Yoo 2007). Here, an *SP* can rely on the QoS requirements of the very applications generating consumer traffic to shift between licensed and unlicensed allocations, and therefore satisfy preferences without any affirmative consumer intervention. As a consequence, the inclusion of service quality as a dimension that is incentive compatible enables these allocation regimes become differentiable products that can generate efficient equilibria.

2.3 Congestion Modeling and Braess's Paradox

John Glen Wardop's (1952) principles for modeling route choice behavior initially addressed congestion modeling for network systems and underlie later contributions by Dietrich Braess. Braess's Paradox (1968), counterintuitively shows that increasing the number of route options—and therefore network capacity—can lead to greater congestion and reduced overall performance. The paradox occurs frequently and in numerous contexts that introduce pricing and choice over congested resources (for an overview, see Steinberg and Zangwill 1987). The communications literature has presented models of selfish routing without pricing (Roughgarden and Tardos 2002; Roughgarden 2005) and based on perceived pricing (Correa et al. 2004). Each example introduces Braess's Paradox through its traditional form—demand-driven consumer responses that increase overall congestion.

We diverge from conventional explanations that describe Braess's Paradox as a consequence of individually dominant strategies and local information. Instead, we demonstrate how service provisioning decisions can exacerbate congestion while maximizing profit in the presence of additional resource competition. The social costs and welfare losses are not directly incurred by the actors themselves (e.g., the *SP*s). Instead, they are borne by the consumer population as an externality.

3 MODEL STRUCTURE

We present a game-theoretic model for wireless broadband service in a market for congestible goods. We describe the strategies available to one or more agents (e.g., *SP*s) in simultaneously determining whether provision service through licensed or unlicensed spectrum and setting prices, as well as their accompanying payoffs. An *SP*'s self-interested behavior leads to a series of unique Nash equilibria with suboptimal social outcomes, despite increasing available resources. This result exemplifies Braess's Paradox—adding resources degrades performance at equilibrium—albeit within the novel form of supplier decisionmaking rather than through consumer responses.

We define a heterogeneous, multi-resource marketplace in which an *SP* may jointly utilize a pair of complementary spectrum allocations with distinct property ownership forms. We assume that an *SP* has been assigned an exclusive allocation through a market-based framework that has a degree of contestability, and where license rights can be reconfigured through secondary markets.[6] We also assume that the *SP* has access to a nearby allocation for unlicensed use. The unlicensed band is structured as a nonexcludable commons that is governed through etiquette constraints on an *SP*'s operations, and may operate on a standalone basis or as a coordinated underlay for incumbent service.[7] Both allocations are bandwidth limited and susceptible to congestion via the cumulative nature of signal interference.

Importantly, we depart from the traditional approach that focuses exclusively on demand-side responses by consumers to congestion. We emphasize supply-side responses, in which *SP*s can manage congestion by deciding whether to use licensed or unlicensed spectrum to offer service to different types of consumers. By modeling endogenous behavior by an *SP* who facilitates information transport, broader considerations reminiscent of market-style mechanisms for impure public goods become relevant, with consumers incurring any interference externalities due to congestion. Our model demonstrates the profit-maximizing actions of a representative strategic agent based on the amount of available spectrum and how competing effects—price competition or congestion-driven service differentiation—may dominate the *modus operandi* for broadband service.

3.1 Service Providers

A finite set *N* of *SP*s act as strategic agents that each individually possess an exclusive license to a spectrum resource and together share access to a second, unlicensed spectrum

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[6] Investment costs are mitigated by the pervasive resale demand for spectrum licenses and the model's assumption that there is free entry into licensed spectrum. For an analysis, see Baumol et al. (1982).

[7] Examples of dedicated unlicensed bands are the Wi-Fi allocations at 2.4 GHz and 5 GHz as well as the recent allocation at 60 GHz. Unlicensed underlays would correspond to opportunistic spectrum designs, such as at 3.5 GHz and 6 GHz.

resource, along with one or more license-exempt providers.[8] For simplicity, we analyze the action of a single, representative SP_i that initially competes for consumers by advertising a flat service price p_i for utilizing its licensed spectrum allocation. We assume that SP_i faces no marginal cost in using its exclusive allocation and that there is a high degree of contestability for licensed resources, mitigating any fixed cost constraints. Similarly, any cost burden SP_i incurs in accessing the unlicensed bandwidth is sunk and faces no marginal cost in provisioning unlicensed service for some group of consumers.[9] In fact, the only cost surrounding unlicensed service is driven by interference externalities and borne equally by all consumers with traffic in the band. Thus, in the absence of any service costs, SP_i does not compete on price for unlicensed service (e.g., $p_i^u = 0$) because any profit-seeking would spur competitive entry to undercut SP_i on price and drive it out of the market.

SP_i serves a group of consumers (x_i) over its exclusive allocation and utilizes available unlicensed spectrum to facilitate service to a second group of consumers (x_i^u). Each of x_i and x_i^u may include one or both of high-elasticity (h) and low-elasticity (l) consumers, meaning: $x_i = x_i^h + x_i^l$ and $x_i^u = x_i^{u,h} + x_i^{u,l}$.

Each of the licensed and unlicensed allocations faces negative congestion externalities borne by interfering radio signals. But, because the SP_i can exclude access to its licensed allocation, congestion in the licensed band is limited to the serviced consumers x_i and notated as

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[8] We assume that each *SP* possesses an equally apportioned sub-allocation within a broader licensed spectrum allocation. This provides a homothetic function where the service population comprises the composite traffic across the set of *SP*s. For an analysis of the theory underlying production functions, see Shephard (1971).

[9] We set $p_i^u = 0$ out of simplicity. This assumption can be extended to a situation where SP_i faces non-zero marginal costs in provisioning unlicensed service, with no qualitative difference. So long as each of the *N SP*s incur the same costs in accessing the unlicensed bandwidth, then the same dynamics exist.

$l(x_i) = \beta x_i$. We note that $l(x_i)$ is a function of the bandwidth of the allocation, the band tier (e.g., low, mid, or high-band spectrum, see GSMA 2021), and the technology implemented in the deployment. Because the unlicensed band lacks excludability, congestion is equally apportioned across the total number of consumers X^u accessing the band for *all SP*s, and is notated as $g(X^u) = \alpha_C X^u$. Under this relation, α_C is an inverse relation of the unlicensed bandwidth (C).[10]

The delivered price for each service consists of the advertised price and congestion costs which impact QoS over the licensed and unlicensed allocations, as shown:

$$\begin{aligned} l(x_i + p_i) &:= \begin{cases} T_1 + \lambda_h \beta(x_i) + p_i \le W_h & \forall\, x_i^h \subseteq x_i \\ T_1 + \lambda_l \beta(x_i) + p_i \le W_l & \forall\, x_i^l \subseteq x_i \end{cases} \\ g(X^u) &:= \begin{cases} T_2 + \lambda_h \alpha_C(X^u) \le W_h & \forall\, x_i^{u,h} \subseteq x_i^u \\ T_2 + \lambda_l \alpha_C(X^u) \le W_l & \forall\, x_i^{u,l} \subseteq x_i^u \end{cases} \end{aligned} \tag{1}$$

Rather than being characterized by service type, these price relations can also be segregated by consumer type, notated as $P_h(Q)$ and $P_l(Q)$, which correspond to the inverse functions of the demand functions $D_h(p)$ and $D_l(p)$, as shown:

$$\begin{aligned} P_h(Q) &= \begin{cases} W_h & 0 \le x_i^h + x_i^{u,h} \le Q_h \\ 0 & \forall\, \{x_i, x_i^u\} \not\subseteq Q_h \end{cases} \\ P_l(Q) &= \begin{cases} W_l & 0 \le x_i^l + x_i^{u,l} \le Q_l \\ 0 & \forall\, \{x_i, x_i^u\} \not\subseteq Q_l \end{cases} \end{aligned} \tag{2}$$

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[10] For simplicity, our model specifies $\alpha_C = \frac{1}{C}$, however, spectral efficiency techniques, such as channel bonding and directional, multi-antenna design could impose a reducing factor in the denominator.

3.2 Consumers

We consider a heterogeneous population with two classes of consumers, Q_h and Q_l, who are willing to contract with SP_i for service according to its posted price p_i. These subpopulations can be defined as $Q_h = \sum_i x_i^h + \sum_i x_i^{u,h}$ and $Q_l = \sum_i x_i^l + \sum_i x_i^{u,l}$, where we assume $Q_h < Q_l$.[11] Each of Q_h and Q_l are distinguished by their sensitivity to congestion, which is based on the service requirements of the applications generating consumer traffic. For example, consumers accessing time-sensitive content, such as video applications, constitute high-elasticity consumers and have a congestion coefficient λ_h. Alternatively, consumers accessing content with little social cost from delay, such as email, constitute low-elasticity consumers and have a coefficient λ_l.[12] Moreover, all consumers in one of the subpopulations Q_h and Q_l share a common valuation for service, denoted by W_h and W_l.[13] Service valuations are based in part on the service requirements of the applications generating demand, therefore $W_h > W_l$.

The serving SP_i jointly selects a representative consumer tuple $\{x_i, x_i^u\}$ which spans a subset of the available consumer population. To account for variability in consumer demand across spectrum bands, we avoid absolute or scaled population inputs and instead define x_i and x_i^u as proportions, where $(x_i + x_i^u) \leq (Q_h + Q_l) = 1$.

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[11] We note that $W_h > W_l$ and $Q_h < Q_l$ and that these assumptions reinforce a boundary constraint for model structure and is reproduced in Corollary 2 of Appendix 4, *infra*.

[12] We assume a unit elastic measure (i.e., $\lambda = 1$) for a perfectly homogeneous consumer market served across N. Thus, $\lambda_h > 1$ and $\lambda_l < 1$ when viewed for all $SP_i \in N$. However, this assumption may be relaxed for boundary instances, such as when λ_h and λ_l are both congestion elastic or both congestion inelastic, but to varying degrees.

[13] W_h and W_l can be more formally notated as: $W_h = \lambda_h(\frac{C_0}{C}) \pm \varepsilon_h$ and $W_l = \lambda_l(\frac{C_0}{C}) \pm \varepsilon_l$ where C_0 is a standard bandwidth metric, such as the customary forward auction allocation size of 70 MHz, and C is the actual bandwidth of the provisioned unlicensed spectrum; ε_h and ε_l are formalized variance values to account for additional considerations.

Demand for wireless service is defined by the functions $D_h(p)$ and $D_l(p)$, which can be expressed as inverse relations of the price functions $P_h(Q)$ and $P_l(Q)$, as noted above. We assume that consumers have perfect information and cannot lower their delivered price by switching providers. Demand for SP_i's service is allocated according to a variant of the Wardop Equilibrium (1952), such that:[14]

$$
\begin{aligned}
&Licensed := &&\begin{cases} T_1 + \lambda_h \beta(x_i) + p_i = P_h(Q_h) & iff\ x_i^h \subseteq x_i \\ T_1 + \lambda_l \beta(x_i) + p_i = P_l(Q_l) & iff\ x_i^l \subseteq x_i \end{cases} \\
&Unlicensed := &&\begin{cases} T_2 + \lambda_l \alpha_C(X^u) = P_h(Q_h) & iff\ x_i^{u,h} \subseteq x_i^u \\ T_2 + \lambda_l \alpha_C(X^u) = P_l(l) & iff\ x_i^{u,l} \subseteq x_i^u \end{cases}
\end{aligned}
\qquad (3)
$$

3.3 Market Framework and Nash Equilibria

We present a simultaneous game where SP_i competes for consumers and maximizes profit by utilizing a pair of instruments—price setting and dictating the amount of traffic it facilitates over its exclusive allocation—as part of an employed equilibrium strategy, as produced in Table 1.

Table 1: Strategic Behaviors for SP_i

Set a price p_i for licensed service.	Select the population x_i that the SP_i serves over licensed resources.

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[14] We assume that both of x_i and x_i^u contain at least on consumer as part of SP_i's provisioned service.

The degree to which the SP_i employs these instruments is based on the size of the unlicensed bandwidth (C) relative to its licensed counterpart and the congestion characteristics within the bands. Through the selected actions, SP_i may achieve a unique Nash equilibrium where:[15]

$$\begin{array}{ll} T_1 + \lambda_h \beta(x_i) + p_i \leq W_h & if\ x_i^h \subseteq x_i \\ T_1 + \lambda_l \beta(x_i) + p_i = W_l & if\ x_i^l \subseteq x_i \\ T_2 + \lambda_h \alpha_C(X^u) = W_h & if\ x_i^{u,h} \subseteq x_i^u \\ T_2 + \lambda_l \alpha_C(X^u) \leq W_l & if\ x_i^{u,l} \subseteq x_i^u \end{array} \qquad (4)$$

The Nash equilibrium may result in suboptimal social outcomes despite maximizing the payoff for the SP_i. For example, in some instances, the SP_i may simultaneously raise licensed service prices and unilaterally offload consumer traffic to the unlicensed bandwidth as a pure strategy. Thus, consumers are subject to higher delivered prices, either via higher advertised prices for licensed service or increased interference externalities from added unlicensed traffic.

3.4 Relevant Outputs: Revenue, Consumer Surplus, and Total Surplus

Based on the observed equilibria, we can calculate a series of output values. Revenue (π_i) for an SP_i is specified as: $\pi_i = p_i x_i + p_i^u x_i^u \Rightarrow p_i x_i$, because we assume that there are no costs in facilitating unlicensed service for SP_i and, as a result, $p_i^u = 0$. We note that this revenue derivation is predicated on our model assumption that consumers have perfect information and there is pricing transparency, meaning that SP_i can advertise a single service price p_i for its licensed service, irrespective of consumer.

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[15] The consumer demand functions are specified so that their inverse relations $P_h(X)$ and $P_l(X)$ are concave; the constraint functions $l(x_i)$ and $g(X^u)$ are convex. This presents a well-defined convex optimization problem with a unique solution. For a mathematical derivation, see Boyd and Vandenberghe (2004).

Consumer surplus (CS_i) is the measured difference between the service valuations (e.g., W_h, W_l) for the classes of consumers Q_h and Q_l, and the delivered price of SP_i's service for its consumer population $x_i + x_i^u$:

$$\begin{aligned} CS_i &= (W_h - \lambda_h(l(x_i) + p_i)) + (\,(W_l - \lambda_l(l(x_i) + p_i) \mid x_i^l > 0) \\ &+ (W_l - \lambda_l g(X^u)) + W_h - \lambda_h g(X^u) \mid x_i^{u,h} > 0)) \end{aligned} \tag{5}$$

For licensed consumers, SP_i's delivered price, $l(x_i + p_i)$, is exclusively determined through SP_i's strategic behavior. For unlicensed consumers, SP_i's delivered price, $g(X^u)$, is determined solely by congestion within the band, including traffic generated by the exogeneous traffic decisions by competing *SP*s.

Total surplus (TS_i), or total surplus, is based on the producer surplus of SP_i, which—given our model assumptions—can be derived from the sum of SP_i's revenue (π_i), and consumer surplus (CS_i) as derived above. As we show in the following Section, total surplus is based on the amount of open-access spectrum available and, in many cases, can counterintuitively cause total surplus to decrease.

4 MODEL DYNAMICS

We present a simultaneous game where an SP_i competes for consumers and analyze the effect of unlicensed spectrum capacity on pricing and service decisions by SP_i. Our analysis focuses on where it is strategically feasible for SP_i to shift consumer traffic between licensed and unlicensed spectrum allocations, the individual payoffs these decisions generate for SP_i, and its welfare impact for consumers.

SP_i may identify a Nash Equilibrium that, depending on the unlicensed bandwidth (C), involves an operating strategy that takes one of three forms:[16]

1. Set an advertised price p_i and uses the licensed band to serve the entire high-elasticity population (Q_h) and at least a portion of the low-elasticity population (e.g., $x_i^l \leq Q_l$).
2. Engage in third-degree price discrimination to serve only the high-elasticity population (Q_h) over the licensed band at a service price p_i.
3. Set an advertised price p_i and uses the licensed band to serve a portion of the high-elasticity population (e.g., $x_i^h \leq Q_h$).

Each strategy is dependent on the degree to which SP_i leverages its right to exclude from its licensed allocation and generate spillovers into the unlicensed commons. Specifically, adding unlicensed capacity may incentivize SP_i to raise prices and reduce the service population for its licensed allocation despite the existence of a broader service alternative. This action is driven by SP_i recognizing that the unlicensed alternative is, in fact, a complement to its exclusive allocation. Thus, SP_i draws on the congestion deterrence of consumers to differentiate its service in a manner that maximizes profit. A summary of the strategies employed and their outputs are presented in Table 2.[17] Each critical point C_i is representative of the minimum unlicensed bandwidth C for SP_i to invoke its locally dominant strategic action.

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[16] *See* Equations 8–12, *infra*. It is implied that SP_i will self-select high-elasticity consumers first based on their higher reservation price (e.g., $W_h > W_l$), serving additional low-elasticity consumers $x_i^l \leq Q_l$ when revenue exceeds marginal cost.

[17] Our summary assumes an initial state at (1)—a mixed service population over licensed resources. Alternate initial states—such as (2) or (3)—correspond to a subset of the summarized strategies. For example, an initial state at (2) would correspond to C_3 within the table. Alternatively, an initial state at (3) would correspond to C_5 within the table and result in strictly price competition.

Table 2: Critical Points for SP_i and Outputs

Unlicensed Bandwidth	Strategic Action		Welfare Impact	
	Price: p_i	Service Population: x_i	Total surplus: TS_i	Consumer surplus: CS_i
C_0	Increase price for licensed service.	Endogenously shift some low-elasticity traffic x_i^l to unlicensed.	Elastic reduction in total surplus TS_i.	Elastic reduction in high-elasticity consumer surplus CS_i^h.
C_1	Reduce price for licensed service and invoke price competition.	Maintain mixed consumer data traffic over license.	Elastic increase in total surplus TS_i.	Elastic increase in high-elasticity consumer surplus CS_i^h.
C_2	Engage in third-degree price discrimination and set the reservation price p_i^{hR} of high-elasticity consumers.	Segregate consumers and exclusively serve high-elasticity subpopulation Q_h over license.	Significant, discontinuous drop in total surplus TS_i.	Complete reduction of high-elasticity consumer surplus CS_i^h. Aggregate consumer surplus CS_i is zero.
C_3	Maintain price for licensed service.	Maintain service subpopulation Q_h over licensed allocation.	Moderately elastic increase in total surplus TS_i.	Moderately elastic increase in low-elasticity consumer surplus CS_i^l.
C_4	Increase price for licensed service.	Endogenously shift some high-elasticity traffic x_i^h to unlicensed.	Moderately elastic reduction in total surplus TS_i.	Moderately elastic decrease in low-elasticity consumer surplus CS_i^l.
C_5	Reduce price for licensed service and invoke price competition.	Maintain high-elasticity consumer data traffic over licensed allocation.	Inelastic increase in total surplus TS_i.	Moderately inelastic increase in low-elasticity consumer surplus CS_i^l. Inelastic increase high-elasticity consumer surplus CS_i^h.

We analyze each of these equilibria, as well as the dynamics described, by presenting an example that derives approximate parameter values from empirical data in the literature (Nevo et al. 2016; Malone et al. 2017). In our example, SP_i's licensed spectrum allocation is fixed, while varying the available unlicensed spectrum bandwidth (C).[18] As a result, any standalone policy decision corresponds to a discrete point within the range of critical points presented, which are indicative of how SP_i will apportion service in response. As shown, SP_i's strategic equilibria produce volatile social outcomes and a sequence of welfare tradeoffs that impact consumer surplus across the heterogeneous population of consumers. We present the unlicensed allocation values that correspond to each critical point (C_i) based on these approximated values, a quantification of each strategic action taken by SP_i at each point, and the absolute and relative impact on social and consumer welfare in Table A1 of Appendix 1.

4.1 Strategies and Equilibria: Service Price and Consumer Population

Depending on the size of the unlicensed spectrum capacity, the SP_i may engage in strategic service differentiation or engage in price competition to combat the expanded market for alternate goods. Figure 1 presents the dynamics between service prices and the licensed service population for a varying unlicensed bandwidth allocation.

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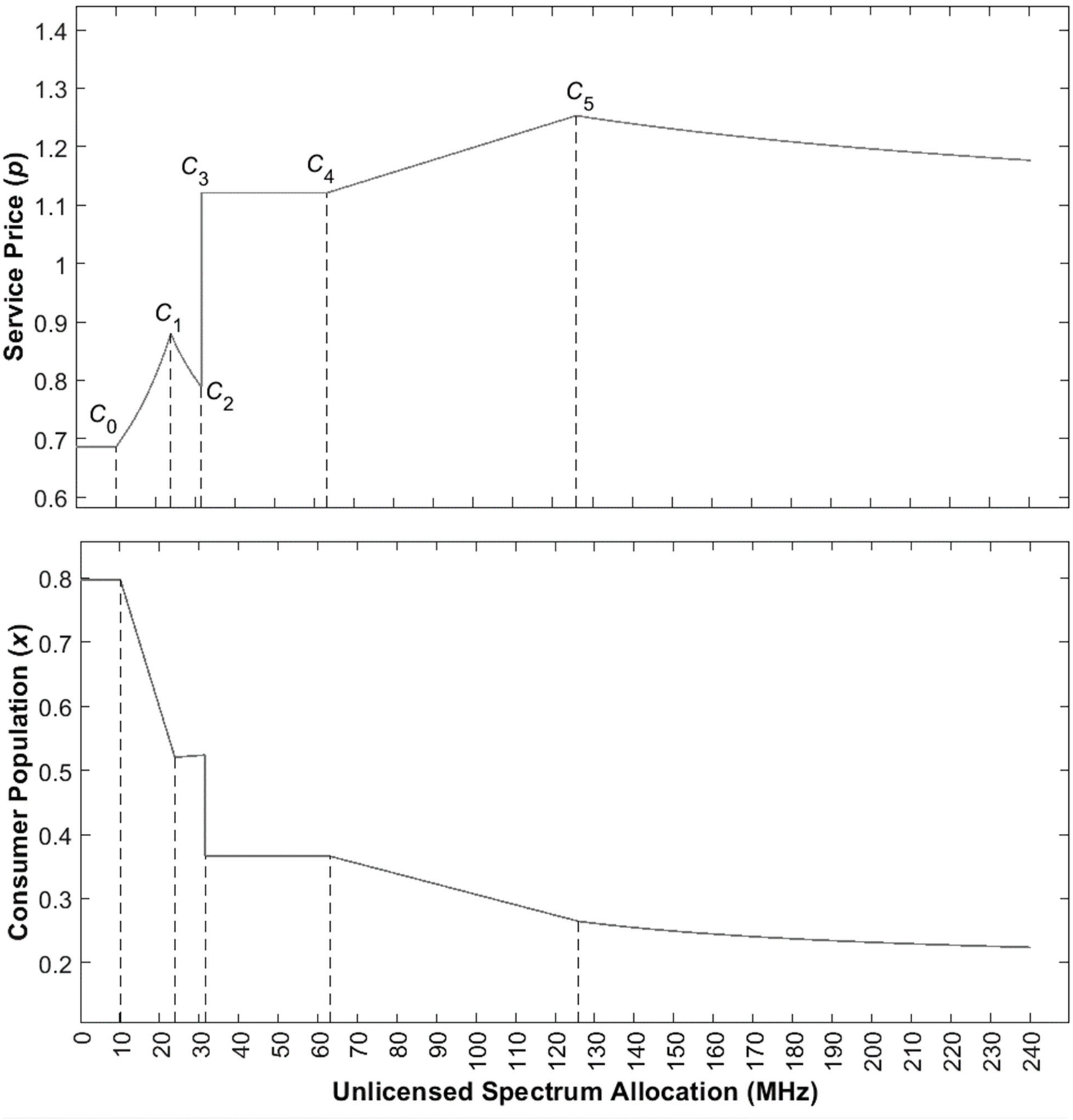
[18] We utilize the following parameters in generating our example:

$$BW_{Licensed} = 70\ MHz;\ W_h = 1.598;\ W_l = 1.115;\ \lambda_h = 1.304;\ \lambda_l = 0.793;\ \beta = 1.0$$

We note that our model is designed to accept, and our results are equally applicable for, differing assignment configurations. While we lack access to precise consumer data, such as congestion elasticity values or valuations for service, we can approximate these values based on anonymized empirical data in the literature. Specifically, Nevo et al. (2016) and Malone et al. (2017) utilize a common dataset containing over 330 million subscriber data observations for a major North American ISP, measured hourly. This data provides an effective indicator of consumer demand elasticity to prices and network congestion, and serves as the source of our estimated elasticity values, λ_h and λ_l, and consumer valuations of service W_h and W_l, presented in the example parameters.

Figure 1: Relationship Between Advertised Service Price and the Percentage of the Consumer Population Served over the Licensed Allocation



In the absence of an available unlicensed commons, SP_i maximizes profit by initiating licensed service for a mixed consumer population x_i at its advertised price p_i. The served consumer base x_i is less than the available consumer population Q, meaning a portion of the low-elasticity subpopulation Q_l does not initially receive service. SP_i abstains from serving these residual consumers because the additional congestion costs $l(x_i)$ of this residual traffic would require an unbalanced reduction in service prices to satisfy the demand functions $D_h(p)$ and $D_l(p)$. SP_i's strategy holds for any initial unlicensed bandwidth allocation below C_0, with the residual consumers receiving unlicensed service via a spectrum commons that is too small to incentivize the SP_i to alter its behavior.

4.1.1 Equilibrium Strategy at C_0

For an unlicensed allocation of C_0, there is sufficient unlicensed bandwidth to present a viable alternative to SP_i's licensed service, particularly for consumers who are congestion inelastic.[19] In response, the SP_i invokes an equilibrium strategy which satisfies the optimization problem below.

$$\begin{aligned}
&\text{Max} \prod_{\{p_i, x_i, X^u\}} p_i\left(Q_h + x_i^l\right) \\
&s.t. \\
&\quad T_1 + \lambda_h \beta(Q_h + x_i^l) + p_i \leq W_h \\
&\quad\quad T_2 + \lambda_l \alpha_C(X^u) \leq W_l \\
&\quad\quad 0 \leq Q_h;\ 0 \leq X^u = (1 - Q_h)
\end{aligned} \tag{6}$$

Intuitively, the presence of additional spectrum resources should spur inter-service competition and lead SP_i to reduce advertised prices p_i to avoid losing its low-elasticity consumers x_i^l. This reaction, however, would fail to account for the differentiability of these services and the heterogeneous congestion sensitivities of consumers. SP_i may instead leverage

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[19] For the approximated values in our example, $C_0 = 10.07$ MHz, as shown in Table A1 of App. 1.

the excludability of its licensed band by invoking a pure strategy of simultaneously raising advertised prices p_i and offloading some low-elasticity consumer traffic x_i^l to the unlicensed band, therefore augmenting the interference externalities borne by unlicensed service traffic X^u. At equilibrium, price and population for licensed service are represented by:

$$\begin{gathered} Q_h + x_i^l = \frac{W_h + (T_2 - T_1) + \lambda_l \alpha_C}{\lambda_h \beta + 2\lambda_l(\beta + \alpha_C)} \\ p_i = (T_2 - T_1) + \lambda_l \left(\alpha_C \big(1 - (Q_h + x_i^l)\big) - \beta \left(\frac{x_i^l}{Q_h + x_i^l} \right) \right) \end{gathered} \tag{7}$$

SP_i's strategy represents a strong correlation between the reduction in consumers receiving licensed service and the price increase. SP_i maintains its differentiation strategy for any unlicensed bandwidth up to C_1,[20] at which point there is enough unlicensed resources to maintain service quality despite additional consumer traffic.

4.1.2 Equilibrium Strategy at C_1

When the amount of unlicensed spectrum bandwidth reaches C_1, SP_i no longer relies on service differentiation as a pure strategy. Because the unlicensed bandwidth has enough capacity to serve as a viable service alternative for SP_i's low-elasticity consumers x_i^l, SP_i must instead engage in price competition to preempt consumer switching. To satisfy the optimization problem in Equation (7), SP_i reduces its advertised price p_i to maintain its mixed service

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[20] For the approximated values in our example, $C_1 = 23.88$ MHz, as shown in Table A1 of App. 1.

population x_i. SP_i maintains engages in price competition for any unlicensed bandwidth up to C_2,[21] resulting in an inelastic price decrease.

4.1.3 *Equilibrium Strategy at* C_2

There is sufficient unlicensed bandwidth at C_2 to serve the entire low-elasticity subpopulation Q_l without degrading performance. In response, the SP_i engages in third-degree price discrimination as part of an equilibrium strategy that satisfies the optimization problem below:

$$\begin{aligned} &\text{Max} \prod_{\{p_i, x_i, X^u\}} p_i(Q_h) \\ &s.t. \quad T_1 + \lambda_h \beta(Q_h) + p_i \leq W_h \\ &\qquad T_2 + \lambda_l \alpha_C(X^u) \leq W_l \\ &\qquad 0 \leq Q_h;\ 0 \leq X^u = Q_l \end{aligned} \tag{8}$$

As part of its pure strategy, SP_i segregates the consumer market based on its distinguishing feature—congestion elasticity (and inversely, price sensitivity)—by excluding any remaining low-elasticity consumer traffic x_i^l from its licensed allocation and tailoring its licensed service to high-elasticity consumers Q_h, who are the most price insensitive. As a result, the SP_i maximizes profit by engaging in willingness-to-pay ("WTP") pricing, where the delivered service price matches the reservation price of congestion-elastic consumers (e.g., $l(x_i + p_i) = W_h$). The equilibrium price and population for licensed service are defined as:

$$Q_h = \frac{(W_h - W_l) + (T_2 - T_1) + \lambda_l \alpha_C}{2(\lambda_h \beta + \lambda_l \alpha_C)} \tag{9}$$

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[21] For the approximated values in our example, $C_2 = 31.56$ MHz, as shown in Table A1 of App. 1.

$$p_i = \frac{(W_h - W_l) + (T_2 - T_1) + \lambda_l \alpha_C}{2}$$

The size of the price increase is strongly correlated with the disparity in congestion elasticities λ_h and λ_l.

4.1.4 *Equilibrium Strategy at C_3*

C_3 is representative of the impact of market segregation by SP_i and its inability to further refine its price discriminatory strategy within consumer subgroups Q_h and Q_l. An unlicensed allocation that larger than C_3 exceeds the minimal channel requirements for transmitting low-elasticity consumer data traffic x_i^l, but remains insufficient for high-elasticity consumers x_i^h. As a result, and in spite of these resource improvements, SP_i maintains its advertised price p_i for the segregated population $x_i^h = Q_h$ despite an expanding resource allocation for alternative service.

4.1.5 *Equilibrium Strategy at C_4*

At an unlicensed bandwidth allocation of C_4,[22] there are enough open-access resources to support high-elasticity consumer data traffic and high-elasticity consumers Q_h may consider service via unlicensed spectrum to be a viable substitute to licensed service. Analogous to its strategic treatment of some low-elasticity consumers x_i^l at lower bandwidths, SP_i invokes an equilibrium strategy that satisfies the optimization problem below:

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[22] For the approximated values in our example, $C_4 = 63.13$ MHz, as shown in Table A1 of App. 1.

$$\begin{aligned} &\text{Max} \prod_{\{p_i, x_i, X^u\}} p_i\left(x_i^h\right) \\ &s.t. \quad T_1 + \lambda_h \beta\left(x_i^h\right) + p_i = \frac{T_2}{2} + \lambda_h \alpha_C(X^u) \leq W_h \\ &\quad \frac{T_2}{2} + \lambda_l \alpha_C(X^u) \leq W_l \\ &\quad 0 \leq x_i^h \leq Q_h \end{aligned} \tag{10}$$

SP_i again leverages its excludability of the licensed band by simultaneously raising its advertised price p_i and offloads some high-elasticity consumer traffic to the unlicensed band. Price and population for licensed service at equilibrium are represented by:

$$\begin{aligned} Q_h - x_i^l &= \frac{W_l + (T_1 - T_2)}{2\lambda_h \beta + (\lambda_h + 2\lambda_l)\alpha_C} \\ p_i &= (T_2 - T_1) + \lambda_h \left(\alpha_C \left(\frac{x_i^l}{1 - (Q_h - x_i^l)} \right) - \beta\left(Q_h - x_i^l\right) \right) \end{aligned} \tag{11}$$

By endogenously offloading additional traffic $x_i^{u,h}$ to the unlicensed band, SP_i capitalizes on its ability to generate interference externalities for the global unlicensed population X^u while bearing only a portion of the costs. And by reducing data traffic within the licensed bandwidth, the SP_i can increase its advertised price p_i through an offsetting reduction to licensed service population x_i so that $l(x_i + p_i) = W_h$. Thus, the SP_i's pure strategy mirrors the strategy at C_0, albeit for a substantially larger unlicensed bandwidth, because of the strong disinclination of high-elasticity consumers to experience congestion.

4.1.6 Equilibrium Strategy at C_5

An unlicensed allocation at or in excess of C_5 provides enough open-access bandwidth to effectively serve high-elasticity consumers Q_h and preempt continued offloading by SP_i.[23] Instead, SP_i reduces its advertised price p_i to compete with available service alternatives and minimizes loss to its service population x_i. The SP_i maintains this strategy for all additional unlicensed bandwidth, mirroring its behavior at C_1, but with a much smaller price reduction being required.

4.2 Payoffs: Service Revenue

As demonstrated above, SP_i's strategic behavior depends on the allocation size of the unlicensed bandwidth. For some allocations, the SP_i's strategy includes simultaneously raising prices and offloading consumer traffic to unlicensed spectrum, thus increasing interference externalities for consumers X^u in the unlicensed band. For others, there is sufficient unlicensed bandwidth for the SP_i to temporarily engage in price competition before reverting to increasingly aggressive forms of service differentiation.

Figure 2 demonstrates that the SP_i's strategic behaviors are pure equilibrium strategies. We model the revenue payoff of SP_i's strategies and provide comparison to the alternate response—competing exclusively on price—with a varying unlicensed bandwidth allocation and overlaid with the critical points presented above. We note that because congestion costs are represented through linearly scaled functions (e.g., $l(x_i)=\lambda\beta(x_i)$) where x_i is decreasing with respect to C, any revenue maximizing action also maximizes profit.

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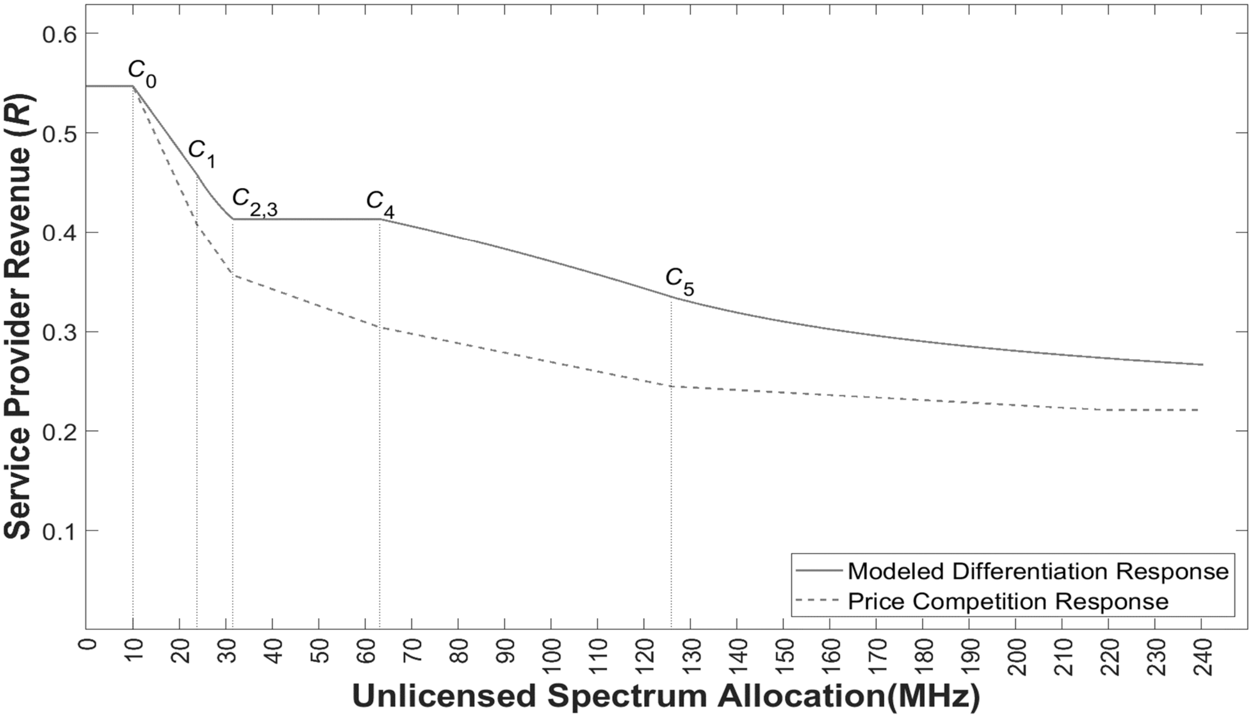
[23] For the approximated values in our example, $C_5 = 23.88$ MHz, as shown in Table A1 of App. 1.

Figure 2: Revenue Comparison Between Service Differentiation and Price Competition



As shown, the SP_i can counteract any increased service competition and minimize its revenue loss by strategically congesting the unlicensed allocation. Because consumers act as both a unit of revenue and a source of congestion, any consumer loss lessens congestion costs, which the SP_i recaptures through higher prices. For the unlicensed allocations between C_0 and C_1; at C_2; and between C_4 and C_5, this congestion-driven service differentiation dominates alternative pricing behavior.

Our results exemplify a fundamental change in competition for broadband service. There is growing practical evidence that consumer congestion elasticities exceed price elasticities for broadband service (Malone et al. 2017), as our model assumes. This evidence introduces an additional dimension to market competition for wireless service, emphasizing service quality as a dimension that supplements quantity and price.

4.3 Welfare Effects: Total Surplus

SP_i's ability to strategically offload traffic according to the amount of complementary unlicensed spectrum enables producer-optimized behavior that, in many instances, is detrimental to system-optimal results (Dafermos and Sparrow 1969; Dafermos 1972). Like observed phenomena in municipal zoning, SP_i maximizes profit by excluding low value uses that place competing demands on their resource budget.[24] As the unlicensed spectrum expands, economic incentives compel SP_i to more aggressively enforce its strategy, generate more spillovers to shared resources, and awaken Braess's Paradox.

The welfare impact (SW_i) of SP_i's strategic equilibria, as well as the critical points C_i corresponding to the minimum unlicensed bandwidth required to trigger SP_i's strategic action, is shown in Figure 3. Overall, the function exhibits significant volatility over a large unlicensed bandwidth (C), including a local maximum and global minimum, two ranges of welfare gain, three ranges of welfare loss, and a discontinuity.

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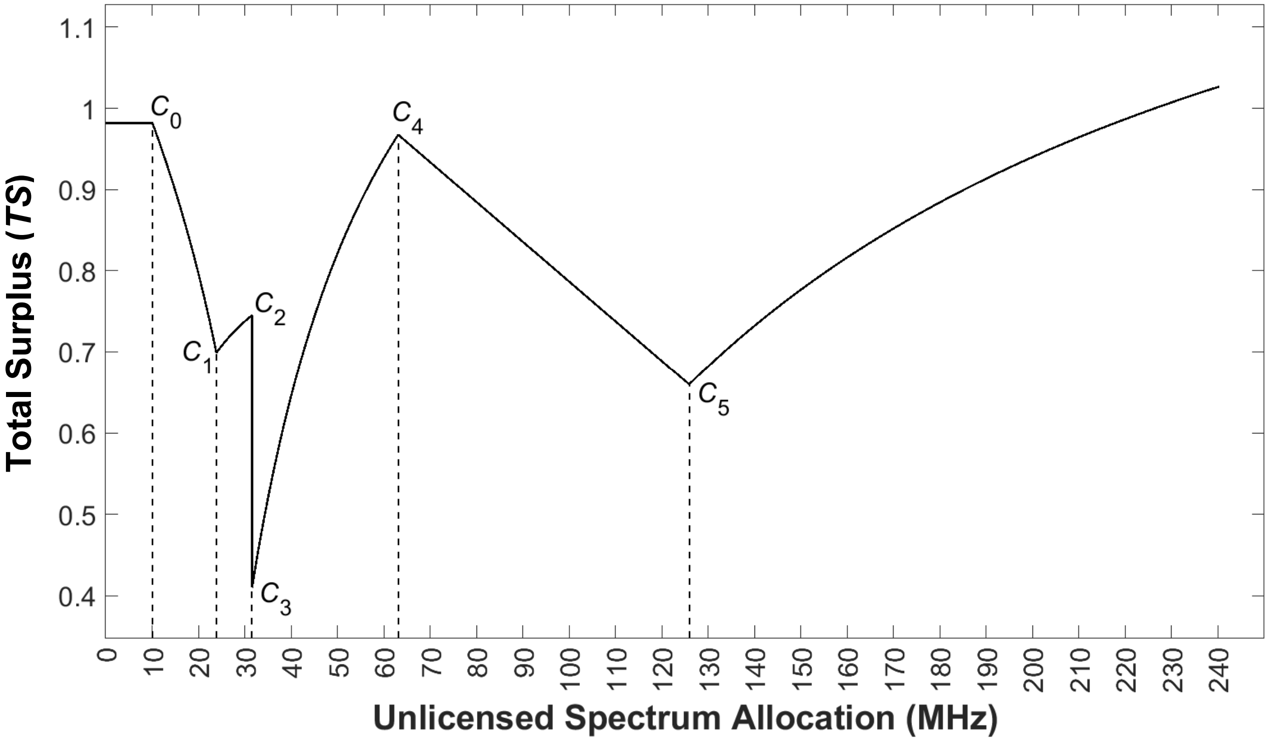
[24] Legal scholars have noted similar exclusions for low-income or shared housing developments in residential zoning ordinances. This is colloquially known as the "Not in My Backyard" ("NIMBY") phenomenon.

Figure 3: Total surplus Effect of Nash Equilibria



The congestion externality associated with shifts of traffic from licensed to unlicensed spectrum gives rise to three internal instances of Braess's Paradox (i.e., at $[C_0, C_1]$, $[C_2, C_3]$, and $C_4, C_5]$), with brief instances of price competition and temporary total surplus recovery interspersed between them. Together, these three Braess's Paradox occurrences give rise to a single, overarching instance of Braess's Paradox where additional unlicensed spectrum causes a reduction in total surplus until the amount of free spectrum far exceeds critical point C_5.

4.3.1 Total Surplus at C_0

At C_0, SP_i invokes its equilibrium strategy of leveraging excludability of its licensed resources and re-provisions service for some of its low-elasticity consumers x_i^l in the unlicensed band. At a component level, SP_i's strategy minimizes loss to producer welfare by reducing its licensed band congestion costs $l(x_i)$ and captures this available surplus through a price increase p_i. This strategic behavior generates an asymmetric reduction in consumer welfare, and therefore a total welfare loss, due to the increased interference externalities imposed on the unlicensed consumer population X^u.[25]

Together, the welfare effects demonstrate Braess's Paradox—a counterintuitive decrease in social outcomes despite an expansion of productive resources. Better conditions lead to poorer outcomes because of self-interested but rational behavior by SP_i that generates locally dominant payoffs in spite of socially suboptimal results. The result is a market failure where SP_i bears few of the interference costs of its decision because congestion is equally apportioned across the total number of consumers X^u. This effect is further exacerbated by the fact that SP_i, rather than the consumers experiencing the congestion, engages in the equilibrium strategy. Self-interested behavior deters SP_i from altering its strategic behavior to promote a better social outcome, and SP_i maintains this strategy for any unlicensed bandwidth capacity up to C_1.

4.3.2 Total Surplus at C_1

For an unlicensed bandwidth of C_1, SP_i's differentiation strategy no longer maximizes profit because the unlicensed bandwidth has enough capacity to compete for service while serving the offloaded consumer traffic. So begins delivered price competition by SP_i to preempt losses of its consumer population to this alternate resource regime. As a result, the confluence of reduced prices within SP_i's exclusive allocation and additional unlicensed capacity to temper

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[25] $\Delta SW_i = 0.28$ (28.85%) for our approximated values, as shown in Table A2 of App. 1. For a description of the values used in our example, and their justification, see note 18, *supra*.

congestion costs $g(X^u)$ generates consumer welfare gain that exceeds SP_i's welfare losses.[26] This competition continues for unlicensed allocations up to C_2.

4.3.3 Total Surplus at C_2

At C_2, SP_i's equilibrium strategy includes engaging in third-degree price discrimination, thereby capturing all available surplus for serving high-elasticity consumers Q_h. Despite maximizing producer surplus, SP_i's strategy results in a significant, discontinuous reduction in total surplus. The welfare losses exemplify a second internal instance of Braess's Paradox that is magnified by the opportunity for price discrimination. Specifically, segregating consumers also minimizes all available consumer surplus in the unlicensed band due to increased interference externalities imposed on the low-elasticity consumers Q_l that comprise unlicensed service X^u.

4.3.4 Total Surplus at C_3

C_3 is representative of the welfare losses incurred by SP_i segregating consumer traffic and engaging in WTP pricing.[27] For an unlicensed bandwidth allocation that exceeds C_3, any additional capacity exceeds the minimum channel requirements for low-elasticity consumer traffic and is realized as consumer surplus. Notwithstanding this improvement, the increase in service capacity remains insufficient to overcome the reduced congestion costs within the licensed band. Nearly all of the prior total surplus losses incurred are exclusively recaptured by the unlicensed population x_i^u.

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[26] $\Delta SW_i = 0.06$ (8.79%) for our approximated values, as shown in Table A2 of App. 1.

[27] This discontinuous drop in welfare corresponds to ΔSW_i = 0.35 (46.05%) for our approximated values, as shown in Table A2 of App. 1.

4.3.5 Total Surplus at C_4

At C_4, SP_i's equilibrium strategy leads higher service price and increased unlicensed congestion so that SP_i can again minimize losses to producer welfare, while reducing consumer welfare. Together, the consumer and producer surplus effects create a third internal occurrence of Braess's Paradox, albeit for high-elasticity traffic.[28] At a mechanical level, SP_i rationally selects an equilibrium strategy that maximizes its payoffs but does not minimize social cost because it does not incur the welfare consequences of its strategies. Rather, these costs are borne by consumers as a congestion externality. The strong inelasticity of SP_i's remaining licensed consumer population x_i^h enables its strategy to remain dominant while the unlicensed bandwidth doubles and consists of a majority of all available spectrum.

4.3.6 Total Surplus at C_5

For an unlicensed bandwidth of at least C_5, there are sufficient open-access resources to require SP_i to compete on price. Nevertheless, the strong disinclination of high-elasticity consumers x_i^h to experience congestion results in an inelastic price response and recovery of total surplus despite these additional resources. Depending on the network conditions, the unlicensed bandwidth for recovering total welfare losses spans a multiplier of two to more than four times the amount of licensed bandwidth. As demonstrated by the welfare dynamics in Figure 3, an unlicensed bandwidth expansion of more than three times SP_i's exclusive allocation is required to completely recoup all losses in total surplus TS_i.

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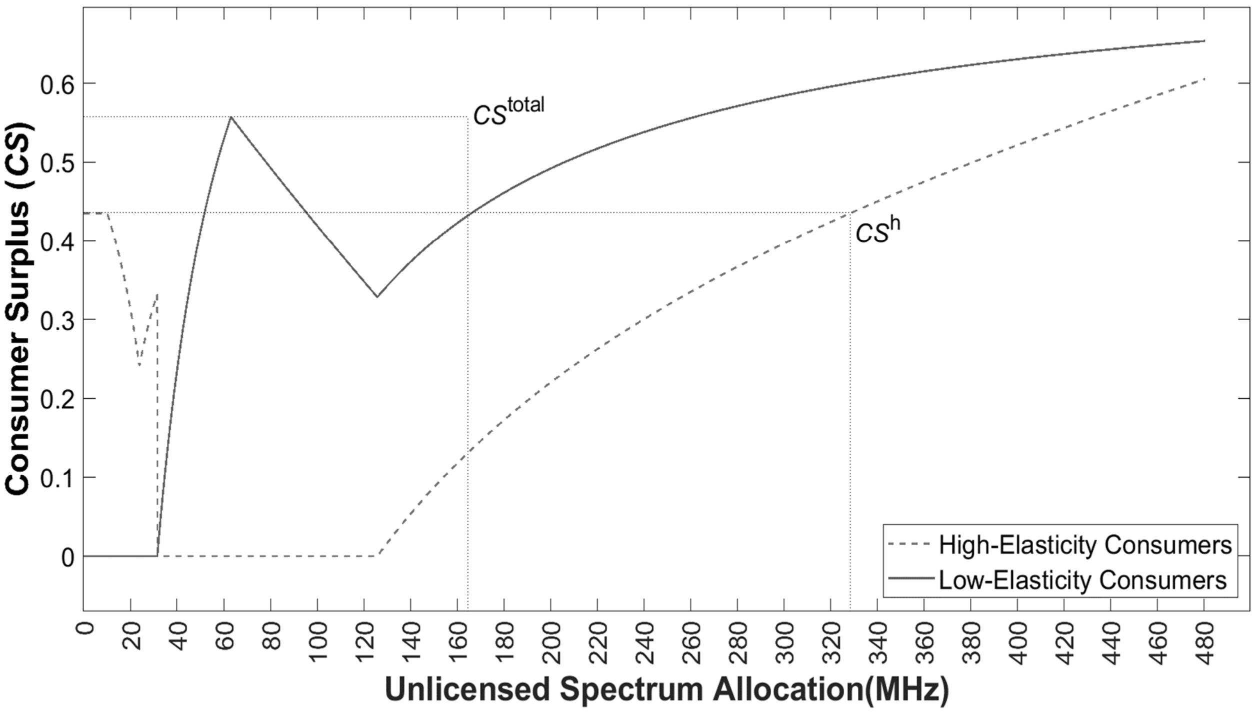
[28] $\Delta SW_i = 0.31$ (31.82%) for our approximated values, as shown in Table A2 of App. 1.

4.4 Welfare Tradeoffs: Consumer Surplus

Considerable debate remains as to whether maximizing total surplus or consumer surplus should be the preferred policy outcome in resource allocation. In neighboring literature, some scholars have supported the measurement of total welfare, under the presumption that any efficiency gains in production are beneficial to society and are available to be passed on to consumers (Williamson 1968; Bork 1978), while opponents instead focus exclusively on consumer surplus, effectively ignoring the need to analyze welfare tradeoffs (Lande 1982; Hovenkamp 2019, 2020). Considering this debate, we supplement our earlier showing of Total surplus (TS_i) with an analysis of consumer surplus.

The debate between maximizing total surplus or maximizing consumer surplus also introduces a dichotomy around how large an unlicensed allocation must be to overcome the volatility and surplus losses that result from SP_i's strategic equilibria. Because producer surplus is a monotonically nonincreasing function, total consumer surplus is recouped faster than total surplus. As shown in Figure 4, recouping all losses in consumer surplus CS_i requires an unlicensed bandwidth that is more than 30% smaller than the allocation required to recoup total surplus losses TS_i.

Figure 4: Allocations for Recouping Consumer Surplus



An interesting feature of the modeled effects is *who* among the heterogeneous consumer population experiences the consumer surplus changes for a given unlicensed bandwidth. Figure 5 presents the consumer surplus tradeoffs between the heterogeneous consumer subpopulations Q_h and Q_l and is overlaid with the critical points presented above to exhibit how consumer surplus varies as unlicensed spectrum capacity increases. As shown, the reductions in consumer surplus are where SP_i invokes a differentiation-dominant strategy (i.e., at C_0, C_2, and C_4), coincide with each internal Braess's Paradox occurrence.[29]

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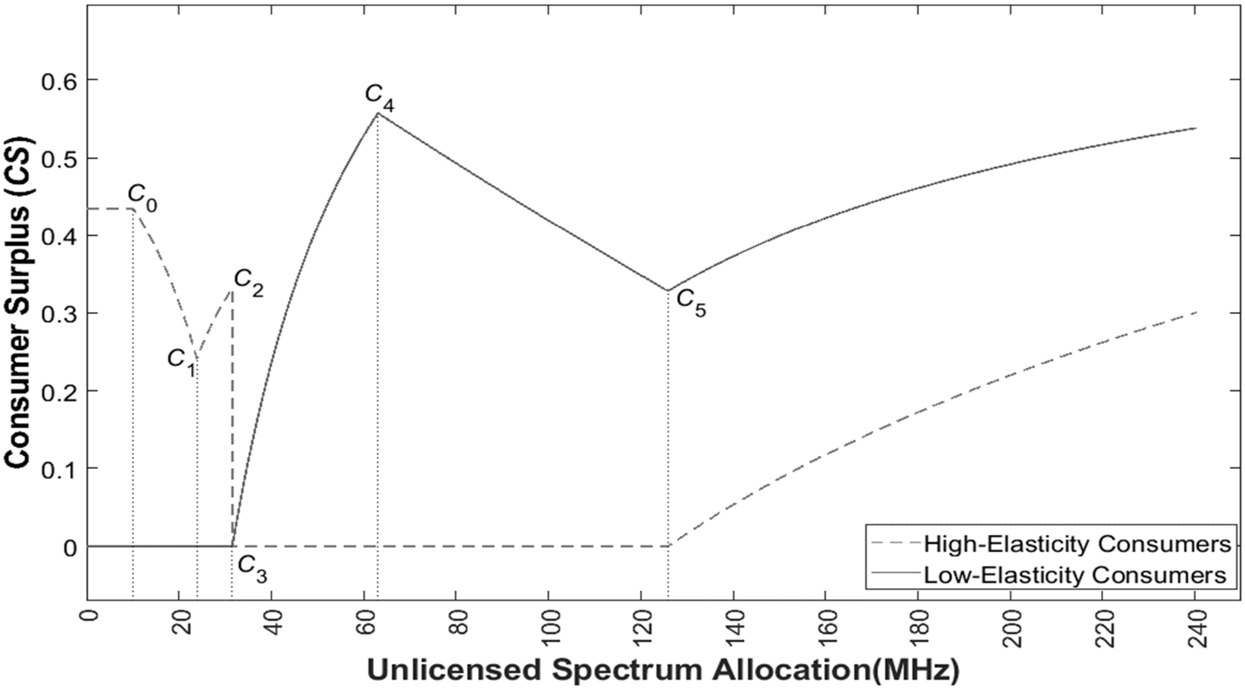
[29] Change in consumer welfare (ΔCS_i) for each consumer subpopulation Q_h and Q_l, based the approximated values provided in our example, is shown in Table A2 of App. 1.

Figure 5: Consumer Surplus Dynamics Between Heterogeneous Consumer Populations



For conservative bandwidths (e.g., [C_0,C_3]), high-elasticity consumers reap all consumer surplus as platform switching is concentrated within low-elasticity consumers. For larger bandwidths (e.g., [C_3,C_5]), the surplus benefits consolidate with low-elasticity consumers as the locus of consumer surplus shifts from the licensed band to the unlicensed band.

From an economic perspective, the consumer surplus dynamics merely consist of a series of wealth transfers. However, as a matter of public policy, these dynamics are of paramount concern in spectrum allocation decisions, particularly given that certain content applications may coincide with different consumer demographics. Moreover, it is becoming increasingly prevalent that high-priority, time-sensitive traffic is the locus for this innovation. From a growth perspective, identifying the necessary unlicensed allocation to recoup high-elasticity consumer surplus CS_i^h may be significant. As Figure 4 shows above, to reach the initial surplus level for high-elasticity consumers Q_h, an unlicensed bandwidth allocation that is over 30% *larger* than the allocation required to recoup total surplus losses TW_i, and more than double the allocation required to recoup total consumer surplus losses CS_i, is required.

5 THE ROBUSTNESS OF THE MODEL

Our motivation is to improve the conceptual framework for spectrum allocation and management. Doing so requires that we can confidently show our modeled results can be replicated for an array of inputs that may correspond to real-world conditions and that the congestion dynamics we identify remain structurally valid.

To these ends, a robustness analysis is presented in Tables A3 and A4 of Appendix 1, spanning 4,487 observations. We vary each input parameter—service valuation (e.g., W_h, W_l) and congestion elasticity (e.g., λ_h, λ_l) inputs for the heterogeneous subpopulations; and general congestion properties of the licensed allocation (e.g., β)—from our initially modeled example based on empirical data, and while complying with the boundary constraints specified.[30] We then test the effects of the modifications to analyze whether the critical points identified in Section 3 remain; for what unlicensed bandwidth SP_i initiates its locally dominant strategies; and the degree to which this behavior impacts price, service, and total surplus, as well as the unlicensed bandwidth required to recoup losses to each welfare metric caused by Braess's Paradox.

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[30] Adjusting the licensed bandwidth allocation $BW_{Licensed}$ does not impact the robustness of the model. Strategic behavior is based on the size of the unlicensed allocation *relative* to the licensed bandwidth. Increasing the licensed bandwidth will result in larger bandwidth values for the critical points described, however the percentages in Table 3 remain the same. *See also supra* note 22. The boundary constraints of the model are presented in Appendix 1, *infra*.

Our robustness analysis demonstrates that the structural validity of our model remains. Specifically, there remains a series of strategic equilibria that maximize individual payoffs but degrade total surplus. Table 3 summarizes the central feature of our analysis for spectrum policymakers—what range of unlicensed spectrum allocations invoke the strategic equilibria and what allocation is required to recoup losses. We present the critical points and their ranges, both in absolute terms and as a percentage of the total bandwidth allocation. Moreover, we specify the recovery points that identify the amount of additional unlicensed spectrum needed to recoup the losses caused by Braess's Paradox, according to the welfare tradeoff discussion above, and present their ranges in absolute terms and as a percentage of total bandwidth.

Table 3: Summary of Modeled Bandwidth Allocation Ranges

Break Point	Unlicensed Allocation (70 MHz Licensed)	Unlicensed Allocation (% Total Allocation)
C_0	0.00—31.52 MHz	0.00%—31.04%
C_1	14.57—35.78 MHz	17.23%—33.82%
C_2	20.10—48.44 MHz	22.31%—40.89%
C_3	20.10—48.44 MHz	22.31%—40.89%
C_4	39.93—84.36 MHz	36.32%—54.65%
C_5	71.11—161.66 MHz	50.39%—69.77%
Recovery Point		
CS_{total}	81.08—322.82 MHz	53.67%—82.18%
SW_{total}	87.92—526.71 MHz	55.67%—88.27%

One interesting component of our robustness analysis is that it yields three potential variants to our original model structure, as previously noted.[31] The variants skew a portion of the allocation ranges listed in Table 3 (e.g., C_0 through C_3) but do not impact the salience of the model. In fact, the variants are addressed as potential features of the model framework in Section 3. We explain each alternate form below and provide a graphical example of the effects on total surplus. Associated price, service population, revenue, and consumer surplus outputs for each variant are in Appendix 2.

First, SP_i may maximize profit by initially serving the entire population (e.g., $Q_h + Q_l$) in circumstances where the high-elasticity population is congestion elastic and the low-elasticity population is highly congestion inelastic (e.g., $\lambda_l \ll 1$). This corresponds to a boundary instance of the initial operating strategy (1), introduced in Section 4, and involves SP_i invoking the strategy at C_0 immediately (e.g., $C_0 = 0$).

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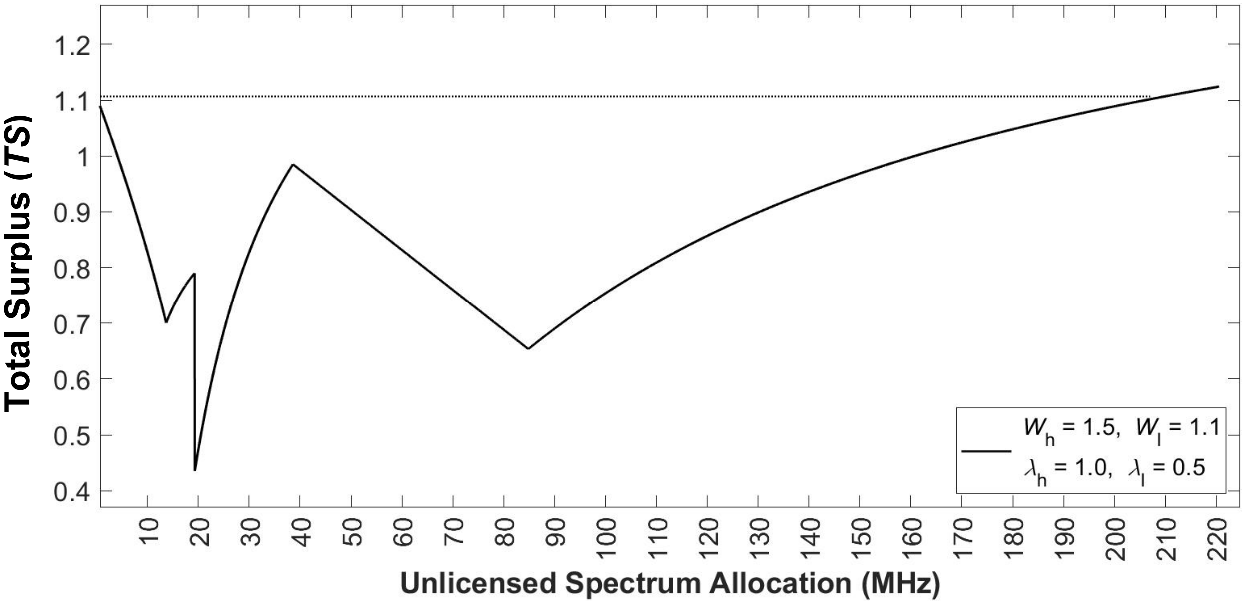
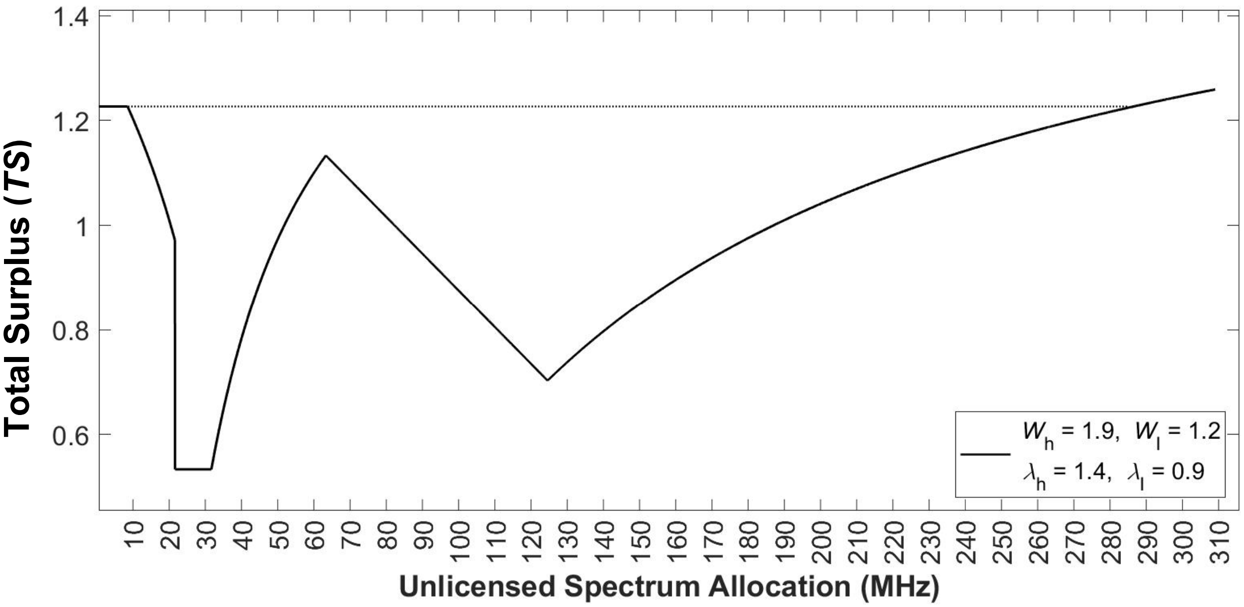
[31] *See supra* note 17.

Figure 6: Total Surplus Effect of Variant: Initial State Serving Entire Population



Second, SP_i may accelerate its service differentiation strategy and invoke third-degree price discrimination for a smaller unlicensed bandwidth. Specifically, where the ratio between service valuation and congestion elasticity is substantially similar across the heterogeneous consumer populations (e.g., $\frac{W_h}{\lambda_h} \approx \frac{W_l}{\lambda_l}$), it is advantageous to accelerate the differentiation. As shown in Figure 7, the strategy outlined at C_1 is obviated entirely and the segregation of consumer traffic serves as the completion of SP_i's strategy at C_0.

Figure 7: Total Surplus Effect of Variant: Accelerated Third-Degree Price Discrimination



Last, for any given set of network conditions, the local maximum for total surplus may occur at the outset (e.g., C_0) or at the consumer surplus peak for low-elasticity consumers (e.g., C_4). An initial state optimum exists where consumers are less price-sensitive and exhibit congestion rigidity. This rigidity occurs where service valuations, and therefore market-clearing prices, exceed congestion elasticities.[32] Alternatively, where high congestion elasticity or conservative service valuations exist, the congestion sensitivity of the high-elasticity population promotes an offloading of service traffic and a local optimum at C_3, as shown in Figure 8.

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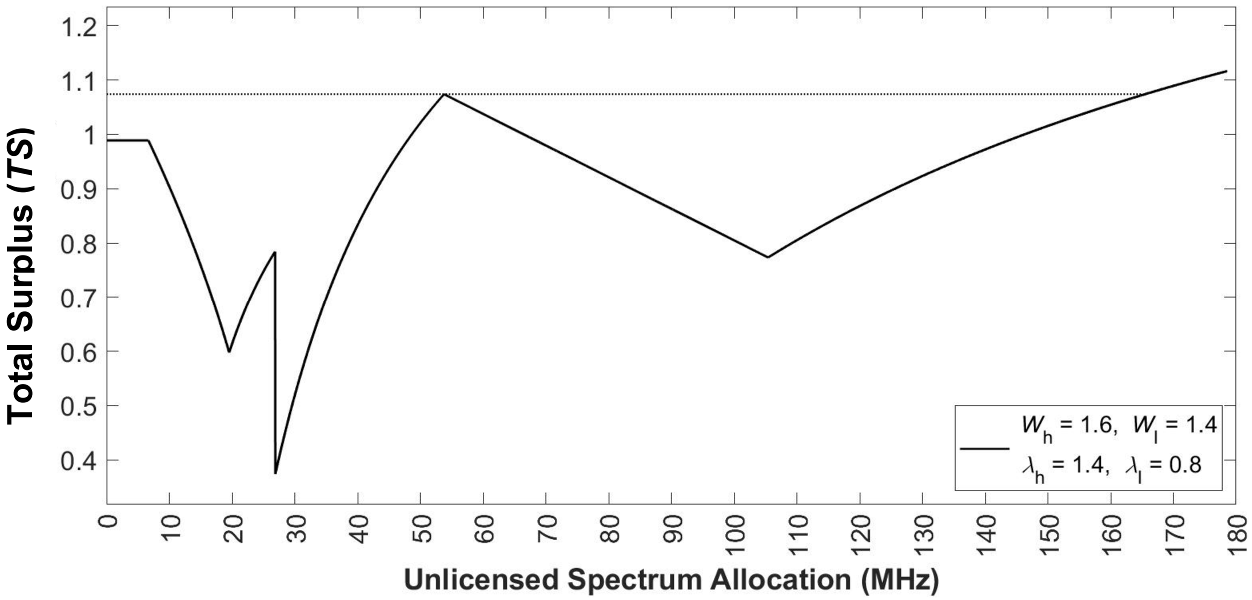
[32] Specifically, where $\frac{W_h}{\lambda_h} > 1$ and $\frac{W_l}{\lambda_l} > 1$. We note that this feature is subject to boundary conditions, described in Corollary 2 of Appendix 4, *infra*.

Figure 8: Total Surplus Effect of Variant: Internal Maximum



Outside of these variants, one additional noteworthy consideration is acknowledging that the largest variance within the robustness statistics is the allocation range required to recoup total surplus or consumer surplus. For a majority of the simulated instances, the multiplier spans values from slightly below three to five times the licensed bandwidth allocation and—in some limiting instances—spans a multiplier of less than one to more than seven times the licensed bandwidth allocation.

6 CASE EXAMPLES AND POLICY ANALYSIS

The FCC has varied the form of property rights that emerge by following different approaches for allocating spectrum to licensed and unlicensed uses. Three recent proceedings exemplify this disparity. Following prior practice for sub-3 GHz spectrum bands, the 3.5 GHz proceeding allocated spectrum to both licensed and unlicensed uses in roughly equal amounts (FCC 2018). The 6 GHz Order designated the entire U-NII-5 through U-NII-8 bands—a 1200 MHz bandwidth—to unlicensed spectrum (FCC 2020c). And the 3.7 GHz Order devoted all of the mid-band spectrum cleared through an incentive auction to exclusive licenses, all to the tune of $81 billion in auction revenue (FCC, 2020a).

The examples are distinct and comprehensive, covering the full range of policy options, while raising important questions for future spectrum policy. They also underlie the policy question that motivates this Article—should the FCC favor polar outcomes in which spectrum is assigned predominantly to licensed or unlicensed uses, or does the FCC have a goldilocks problem in administering spectrum resources?

We apply our model to the allocations listed and analyze the impact of the congestion dynamics inherent within each policy decision. While a myriad of broader considerations (e.g., revenue generation, innovation potential, market access, etc.) underlie each decision and remain hotly debated by policymakers, they are each dependent on exogenous social factors that are beyond the scope of this Article. Our focus is on a crucial feature that is solely driven by the incentive frameworks of the spectrum rightsholders themselves.

Our procedural framework is as follows. First, we collect publicly available auction data or filings from FCC proceedings to estimate initial input parameters for service valuation, congestion elasticity, and band properties. Our collection includes scaling the collected values to historical data from nearby spectrum blocks. Second, we utilize our robustness framework to vary the estimated inputs and run a series of iterations for our model to account for approximation errors.[33] Third, we present the unlicensed spectrum values and social output metrics, along with their standard errors, corresponding to the strategic equilibria of our model.

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[33] We imposed a measurement range of ± 20% to each input variable and ran the model over 1,200 iterations. The described ranges serve as upper and lower bounds.

Finally, we analyze the counterfactuals for each instance to examine what would have happened if the FCC had followed a different policy design.

While FCC has a continuum of allocation alternatives at its disposal, we limit our analysis to the three most common paradigms—all unlicensed, splitting the allocation, and all licensed. Table 4 summarizes the output metrics of each spectrum policy decision we analyze along with counterfactuals, when applied to our model design.[34] The FCC's deployed implementation in each case is marked in bold.

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[34] For each *All Licensed* framework, we include a measure of the amount of additional unlicensed spectrum needed to match the licensed design's welfare output; the values are denoted with an asterisk. For each *All Unlicensed* framework, we include a range representative of the simulated instantiations, except for the FCC's actual implementation at 6 GHz.

Table 4: Summary of Modeled Outputs for Case Examples (Actual Outcome in Bold)

Spectrum Band	Available Spectrum for Allocation	Allocation Design	Total Surplus	Consumer Surplus	Unlicensed Spectrum Needed to Avoid Braess's Paradox
3.5 GHz	150 MHz	All Licensed	1.32	0.55	558.27 MHz*
		Equally Split	**0.65**	**0.35**	253.71 MHz
		All Unlicensed	0.55–0.59	0.22–0.31	369.61–1774.13 MHz
3.7 GHz	280 MHz	All Licensed	**2.05**	**0.73**	859.07 MHz*
		Equally Split	1.29	0.55	352.79 MHz
		All Unlicensed	1.00–1.86	0.32–1.38	306.81–828.39 MHz
6 GHz	1200 MHz	All Licensed	1.02	0.50	2799.05 MHz*
		Equally Split	0.96	0.30	1841 MHz
		All Unlicensed	**1.30**	**0.96**	1007.78 MHz

6.1 3.5 GHz Proceeding (CBRS)

The FCC's framework for 3.5 GHz spectrum ("CBRS") represented a landmark initiative for spectrum sharing—a three-tiered framework for opportunistic commercial access to spectrum used by the U.S. Navy and Fixed Satellite Service systems. As structured, the framework allows up to 70 MHz of exclusive access to the lower portion of the band, provided that each licensee give way whenever the federal incumbents need access. The remaining 80 MHz is available to all entities on an unlicensed basis, including as a secondary access regime for commercial licensees (FCC 2018). The result was a roughly equal allocation to licensed and unlicensed spectrum.

6.1.1 Current Implementation

Relying on public auction data, we approximate a series of input values for the FCC's split allocation and run it through our model framework. Each input is derived from a multivariate calculus that includes the number of bidders, the ratio of attained-to-available licenses, and the price calculated in MHz/POP, is scaled by comparable auction data for mid-band spectrum proceedings over the last two decades, and is run through our robustness framework.[35] Table 5 presents the unlicensed bandwidth for each Nash Equilibrium and the minimum unlicensed bandwidth required to recoup all losses in consumer surplus CS_{total} and all losses in total surplus TS_{total}, along with the welfare outputs and standard errors for each.

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[35] Based on our survey, both the median service valuations for exclusive spectrum resources and the congestion elasticities of consumers are moderate. Our calculations generate the following inputs: $BW_{Licensed} = 70$; $W_h = 1.202$; $W_l = 0.760$; $\lambda_h = 1.10$; $\lambda_l = 0.525$; $\beta = 0.88$

Table 5: Summary of Modeled Bandwidth and Output Values: 3.5 GHz

Break Point	Unlicensed Allocation	Total Surplus	Consumer Welfare
C_0	9.30 MHz (6.76)	0.73 (0.20)	0.32 (0.12)
C_1	21.88 MHz (4.30)	0.55 (0.16)	0.21 (0.11)
C_2	30.11 MHz (5.23)	0.61 (0.08)	0.27 (0.03)
C_3	30.11 MHz (5.23)	0.31 (0.13)	0.00 (0.00)
C_4	60.19 MHz (10.47)	0.69 (0.14)	0.38 (0.06)
C_5	136.60 MHz (22.77)	0.52 (0.07)	0.25 (0.06)
Recovery Point			
CS_{total}	198.00 MHz (26.70)	0.63 (0.14)	0.38 (0.15)
SW_{total}	253.71 MHz (43.27)	0.73 (0.17)	0.53 (0.19)

The equal-allocation approach taken by the FCC falls between critical points C_4 and C_5, and results in sub-optimal welfare outcomes SW_i = 0.65; CS_i = 0.35. From a congestion modeling perspective, appeasing both licensed and unlicensed stakeholders results in Braess's Paradox and detrimental outcomes for consumers. Specifically, for each 10 MHz license block that the FCC auctioned off for exclusive use to complement the 80 MHz commons available to a licensee, an additional 28 MHz of unlicensed spectrum is necessary to recoup consumer welfare and an additional 36 MHz to overcome total surplus losses. Using the auction data provided to estimate demand, the opportunity cost of procuring this free spectrum to recover welfare is nearly $210,000 per license. When aggregated over the more than 22,000 licenses auctioned, these costs exceed $4.6 billion, which is more than the total auction revenues of the FCC's 70 MHz licensed allocation.

6.1.2 Policy Alternatives

What would have happened if, instead of splitting the spectrum between licensed and unlicensed uses, the FCC had adopted one of the other, more polar allocation possibilities? We consider first the counterfactual scenario in which all 150 MHz of spectrum available within the 3.5 GHz band were allocated to unlicensed uses and examine whether this allocation is large enough to promote welfare gains. Because a fundamental assumption of our model is that strategic agents endogenously shift traffic into the unlicensed band, we must identify a coexisting licensed allocation to model these dynamics. We consider three instantiations of our congestion model based on nearby licensed spectrum, beginning with adjacent licensed spectrum at 3.45 GHz and then expanding to include the 3.7 GHz and 2.5 GHz bands, respectively. We present the critical points corresponding to an agent's strategic equilibria in Table A5 of Appendix 3.

Overall, our analysis shows that a shift to open-access spectrum in the 3.5 GHz band does not remedy the shortfalls of the current implementation and, in fact, exacerbates the effect in each of the instances analyzed. For each of the considered licensed allocations with access to this hypothetical spectrum commons, welfare outcomes are worse than what is generated in the split allocation. For example, in the instance where only 3.45 GHz licenses are complementary to the band, the resulting values are SW_i = 0.59; CS_i = 0.31. A unified framework with 3.45–3.7 GHz licensees results in values SW_i = 0.57; CS_i = 0.25. And an access regime where licensees from the 2.5–3.7 GHz bands utilize the unlicensed bandwidth results in values SW_i = 0.55; CS_i = 0.22. All of these measures fall below those obtained by the FCC's decision to split the spectrum between licensed and unlicensed uses, which yielded SW_i = 0.65; CS_i = 035.

Second, we estimate what would have occurred had the FCC used auctions to assign all of the spectrum in the 3.5 GHz proceeding to licensed uses. As Table 5 demonstrates, a standalone 70 MHz licensed allocation provides a local total surplus maximum (e.g., SW_i = 0.73). The purpose of our counterfactual is to estimate the additional welfare gains or opportunity costs inherent in licensing the entire 3.5 GHz band, rather than a portion. Recent auction data for the adjacent 3.45 GHz band serves as an effective proxy (FCC 2021) in that both bands occupy adjacent portions of the spectrum and thus share similar propagation characteristics, and both impose service rules that include coordination with federal incumbents.[36] Using auction data for the 3.45 GHz band, we can approximate consumer demand for a hypothetical auction of the full 3.5 GHz band and run our analysis.[37]

The alteration in service valuations suggest allocating all of the spectrum to licensed uses would have yielded expected welfare outputs of SW_i = 1.32; CS_i = 0.55. These outputs would have exceeded those resulting from the FCC's actual decision to split the spectrum between licensed and unlicensed uses, which produced SW_i = 1.32; CS_i = 0.55. Thus, according to the approach we describe, allocating all CBRS spectrum to licensed uses resources would have outperformed the current design along both measures.

In short, allocating all of the spectrum available in the 3.5 GHz proceeding to licensed uses would have provided a better outcome in terms of total surplus and consumer surplus than

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[36] The equivalence we suggest has some limitations. Licenses in 3.45 GHz were issued for larger Partial Economic Areas ("PEAs") rather than the CBRS county licenses and fewer licenses are encumbered by federal operators. Despite these advantages, the 3.45 GHz band has stricter technical limitations, resulting in greater equipment and buildout costs.

[37] Based on auction data for the 3.45 GHz band, there exists a high median service valuation for exclusive spectrum resources and congestion elasticities remain moderate, given similar technical rules and physical characteristics for the bands. This leads to solely a change in service valuations where: $BW_{Licensed} = 150$; $W_h = 1.758$; $W_l = 1.109$; $\lambda_h = 1.10$; $\lambda_l = 0.525$; $\beta = 0.88$

the one actually obtained by splitting the spectrum across licensed and unlicensed uses or had it allocated all of this spectrum to unlicensed uses.

6.2 3.7 GHz Proceeding (C-Band)

The swath of spectrum between 3.7 and 4.2 GHz ("C-Band") was first authorized for fixed satellite communications. Following strong congressional support, the FCC relocated the satellite incumbents and auctioned the lower 280 MHz of "prime mid-band" spectrum entirely for licensed uses (FCC, 2020a). The result was the most valuable spectrum auction in history, generating $81 billion in revenue.

6.2.1 Current Implementation

Based on public auction data, we approximate service valuations and overall consumer demand for C-Band spectrum as applied to our model. Again, each input is derived from the multivariate calculus specified above, scaled based on historical data for comparable proceedings, and run through our robustness framework.[38] When placed in our model, an exclusively licensed allocation suggests, on average, expected welfare outputs of SW_i = 2.05; CS_i = 0.73. The model also indicates that, from a congestion perspective, operators maximize profit by serving the entire available service population.

6.2.2 Policy Alternatives

We explore counterfactuals assessing the impact on total surplus and consumer surplus had the FCC pursued one of its two other primary allocation options. First, we consider the

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[38] Based on our survey, there exists a very high median service valuation for exclusive spectrum resources and moderate congestion elasticities, akin to other mid-band frequencies which have been allocated. Our calculations generate the following example inputs: $BW_{Licensed} = 280$; $W_h = 2.694$; $W_l = 1.720$; $\lambda_h = 1.353$; $\lambda_l = 0.689$ β = 1.05.

alternative where the band is split equally between exclusive licenses and an unlicensed commons. Because the FCC provides individual statistics for each block within its allocation, we recalibrate our service valuations by arbitrarily taking the weighted average of the net proceeds for the lower 140 MHz of spectrum (i.e., half of the band allocation) and scale the service valuations, resulting in substitute values $W_h = 2.989$; $W_l = 2.016$ for our example. We present the critical points for an agent's strategic equilibria, the welfare outputs they generate, and the standard errors for each in Table 6.

Table 6: Counterfactual Bandwidth and Output Values: 3.7 GHz Split Allocation

Break Point	Unlicensed Allocation	Total Surplus	Consumer surplus
C_0	0.00 MHz (9.06)	2.34 (0.11)	0.71 (0.16)
C_1	18.35 MHz (6.68)	1.64 (0.14)	0.51 (0.12)
C_2	26.01 MHz (7.58)	1.73 (0.12)	0.60 (0.07)
C_3	26.01 MHz (7.58)	1.06 (0.17)	0.00 (0.00)
C_4	52.02 MHz (15.62)	2.08 (0.06)	1.01 (0.07)
C_5	116.55 MHz (28.23)	1.07 (0.11)	0.43 (0.06)
Recovery Point			
CS_{total}	314.13 MHz (19.78)	0.63 (0.08)	1.01 (0.19)
SW_{total}	352.79 MHz (55.46)	2.34 (0.14)	1.64 (0.11)

Our findings suggest that splitting the C-Band spectrum between licensed and unlicensed uses would have imposed significant congestion costs that would have reduced the expected welfare outputs. On average, an additional 174.13 MHz of unlicensed bandwidth beyond the hypothetical 140 MHz available under a split allocation would have been required to recapture the lost consumer surplus and an additional 212.79 MHz is required to recoup total welfare losses caused by Braess's Paradox. Referencing the auction data for C-Band spectrum, the opportunity cost of this additional spectrum is over $14,250,000 per license, or roughly a total of over $57.5 billion.

Second, we consider what would have occurred had the FCC opened the entire C-Band as an unlicensed spectrum commons. As noted above, the congestion dynamics replicated in our model assume the existence of a corresponding licensed allocation, which we demonstrate through three hypotheticals: offloading from the nearby 3.45 GHz band; a complementary regime that includes traffic from the licensed portion of the CBRS band; and extending offloading from sub-3 GHz spectrum, specifically the recently assigned spectrum licensees at 2.5 GHz. We present the critical points corresponding to an agent's strategic equilibria in Table A6 of Appendix 3.

A completely unlicensed C-Band reinvigorates the debate in the literature involving welfare tradeoffs.[39] In the case of offloading from the 3.45 GHz band, allocating all of the spectrum to unlicensed uses would have reduced total surplus from TS_i = 2.05 to TS_i = 1.86 but would have increased consumer surplus from CS_i = 0.73 to CS_i = 1.38. When including the licensed portions of the CBRS band, allocating all of the spectrum to unlicensed uses would have

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[39] See Section 4.3, *supra*.

reduced total surplus from TW_i = 2.05 to TW_i = 1.47 but would have increased consumer surplus from CS_i = 0.73 to CS_i = 0.92. And where licensees from the 2.5 GHz to 3.5 GHz bands utilize the newly unlicensed spectrum, allocating all of the spectrum to unlicensed uses would have reduced total surplus from TS_i = 2.05 to TS_i = 1.00. Unlike the other cases, it would also have reduced consumer surplus from CS_i = 0.73 to CS_i = 0.32.

Overall, these dynamics demonstrate that determining the optimal spectrum allocation may require more nuanced analysis. The decision to allocate all of the C-Band spectrum to licensed uses performed better on both welfare metrics than would have a decision to split the spectrum between licensed and unlicensed uses. The same is true for the broad instance of allocating all of the spectrum to unlicensed uses. The implications of the narrow and moderate cases of devoting all of the spectrum to unlicensed uses are more complex. Our model suggests that they would have reduced total surplus but increased consumer surplus. As such, the optimal policy outcome depends on which welfare metric is preferred.

6.3 6 GHz Proceeding (Wi-Fi 6E)

The FCC's 6 GHz rulemaking approved the largest expansion of unlicensed bandwidth in history, allocating 1200 MHz in spectrum to an open-access commons. The decision was a surprising paradigm shift in spectrum design, not just because of the size of the allocation but also because of the decision not to allocate any spectrum to licensed bandwidth. On an international level, countries have failed to align on how to best allocate this band, raising questions as to whether the United States is at the forefront in modernizing spectrum policy or making ill-advised judgments.[40]

6.3.1 Current Implementation

We analyze the impact of the FCC's decision to devote all of this large swath of spectrum to unlicensed uses. This spectrum is expected to be used for next-generation Wi-Fi. Because no data is available to approximate demand or price sensitivity, we rely on the economic value metrics included in public filing disclosures to the FCC (CTIA 2019; NCTA 2020; Wi-Fi Alliance 2020) and congestion reporting for the neighboring Wi-Fi allocation at 5 GHz (Dynamic Spectrum Alliance 2021). Given the breadth of the allocation, we assume the 6 GHz band serves as a complementary spectrum rights regime to exclusive mid-band licenses from 3.45 to 3.7 GHz, corresponding to 450 MHz of unified licensed spectrum.

Based on public disclosures, we estimate a series of input values and model a conservative circumstance where the 6 GHz allocation serves as a standalone unlicensed allocation.[41] Table 7 presents the estimated critical points and the welfare outputs that result from the congestion routing decisions of one or more operators, along standard errors for each.

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[40] For example, much of Latin America, South Korea, and Saudi Arabia have followed the U.S. position. Europe has split the band, releasing the lower portion for Wi-Fi and proposing licenses for the upper portion. China plans to license the entire band for cellular services.

[41] Based on our review, there exists a relatively low median service valuation for exclusive spectrum resources and high congestion elasticities, particularly for time-insensitive applications. Our calculations generate the following example inputs: $BW_{Licensed} = 450$; $W_h = 2.143$; $W_l = 1.555$; $\lambda_h = 1.740$; $\lambda_l = 0.860$; $\beta = 1.20$.

Table 7: Summary of Modeled Bandwidth Allocation Ranges: 6 GHz Unlicensed

Break Point	Unlicensed Allocation	Total Surplus	Consumer surplus
C_0	94.10 MHz (21.98)	1.16 (0.25)	0.46 (0.15)
C_1	129.74 MHz (17.57)	0.86 (0.22)	0.26 (0.14)
C_2	172.76 MHz (20.50)	0.94 (0.11)	0.51 (0.06)
C_3	172.76 MHz (20.50)	0.48 (0.14)	0.00 (0.00)
C_4	345.56 MHz (41.04)	1.26 (0.15)	0.78 (0.05)
C_5	745.38 MHz (96.27)	0.93 (0.10)	0.57 (0.11)
Recovery Point			
CS_{total}	928.43 MHz (109.54)	1.13 (0.17)	0.78 (0.15)
SW_{total}	1007.78 MHz (124.26)	1.26 (0.22)	0.67 (0.17)

Our findings indicate that releasing the entire 6 GHz bandwidth for next-generation Wi-Fi appears to mitigate congestion costs. The 1200 MHz allocation provides more than enough capacity to overcome Braess's Paradox, recoup any transitory surplus losses, and achieve a socially beneficial result where SW_i = 1.30; CS_i = 0.96.

6.3.2 Policy Alternatives

We analyze a pair of policy alternatives where the 6 GHz band includes a licensed bandwidth allocation. The counterfactuals seek to determine whether the congestion characteristics we specify support such a drastic expansion of unlicensed bandwidth.

First, we consider a split 6 GHz band allocation containing an equitable allocation of licensed and unlicensed bandwidth and adjust $BW_{Licensed}$ to 600 MHz.[42] When placed within our congestion model and scaling to the adjusted bandwidth allocation, the parameters suggest that a split allocation imposes significant detriments on welfare. The resulting output values, $SW_i = 0.96$; $CS_i = 0.30$, fall within the second Braess's Paradox instance between critical points C_4 and C_5.

Second, we extend our counterfactual to simulate a comprehensive, licensed allocation over the entire 1200 MHz of available spectrum. Because exclusive license holders have the option of determining when and for whom they provide service, general solicitations of economic benefit are ineffective valuation tools; auction data provides a more accurate representation.[43] However, because no nearby bands or extraterritorial auction proceedings exist to provide a value indication, we instead approximate service values by estimating a reserve price to clear microwave incumbents in the band, which serves as serves as an effective lower bound for valuing service.

According to market research reports (GSMA 2021) and facts raised in a recent D.C. Circuit decision (*AT&T Services, Inc. v. FCC*, No. 20-1190, 2021 U.S. App. LEXIS 38370 (Dec. 28, 2021)), the point-to-point microwave services concentrated in the 6 GHz band have been valued at $15.5 billion (GSMA 2021).[44] Importing this value into our framework and

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[42] We note that this is the approach suggested in several of the filing disclosures mentioned, and impliedly assumed in the solicited valuations (CTIA 2019).

[43] An appropriate example of this dynamic is in TV Broadcast, where licensees have argued inflated economic values that were quickly debunked by market mechanisms, such as the 700 MHz auction.

[44] The only projected auction revenue we are aware of valued the 6 GHz band at $22+ billion.

scaling it to auction proceedings over the last two decades, we model a hypothetical exclusive allocation of the 6 GHz band.[45]

When run through our congestion model, the exclusively licensed allocation produces expected welfare outputs of SW_i = 1.02; CS_i = 0.50. As a result, exclusive licensing of the band is suboptimal to the current allocation. The unlicensed bandwidth allocation is significant enough to impose price competition in the band and promote beneficial channel conditions which exceed an exclusive licensing framework.

6.4 A Policy for the Future

When evaluated in aggregate, our analysis provides a useful framework for analyzing congestion pricing and identifying common themes across recent policy actions. The unified thesis that emerges from each case study is that bold strokes are necessary to achieve the highest and best use of the spectrum. Attempting to strike middle ground among stakeholders is often imprudent when considering the economics of congestion. Although our model identifies internal points that may be optimal, they are infrequent, and determining their precise location will likely prove intractable. Thus, while a perfect balance may exist but it is encompassed by strategic equilibria where rational, efficient market participants maximize payoffs by imposing congestion externalities on consumers. Instead, we suggest that policymakers go all in on the most economically beneficial allocation.

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[45] Our example uses the following inputs: $BW_{Licensed} = 1200$; $W_h = 1.90$; $W_l = 1.212$; $\lambda_h = 1.740$; $\lambda_l = 0.860$; β = 1.05.

We provide some guideposts for determining which allocation—licensed or unlicensed—is most appropriate under the circumstances raised by our congestion model. Each of these variations is a crucial consideration in determining the most efficient spectrum allocation design.

First, policymakers must take stock of the size of the spectral resources that are at their disposal. Many modern spectrum allocations require relocating incumbents, clearing the band, and identifying new technical limitations that ensure novel operations can coexist with neighboring band deployments. These elements each impose upfront fixed costs at diseconomies of scale. As indicated within our summary statistics, an unlicensed allocation requires, on average, 297% more bandwidth to achieve the same total surplus gains as licensed spectrum, when accounting for congestion characteristics.[46] Furthermore, for unlicensed allocations, these costs cannot be recouped through auction revenues and must instead be discounted through future growth. As a result, policymakers must decide whether the opportunity costs of clearing nearly three times bandwidth are excessive.

Second, a policy design must recognize the cumulative nature of spectrum use and identify where interoperability exists between bands. This is apparent in two regards. Licensed allocations forgo any consumer surplus gains from integrating outside spectral bands. Unlicensed allocations must be mindful of what bands potentially offload into the allocation and if additional unlicensed complements exist. Both components factor into identifying the necessary bandwidth to overcome socially detrimental strategic actions.

Third, the magnitude of, and disparity between, congestion elasticities and service valuations will impact the efficacy of policy designs. These factors also frame what bandwidth

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[46] *See infra*, App. 3.

allocations coincide with the critical points referenced in the model. For example, in instances where service valuations exceed congestion elasticities or general band characteristics suggest decent propagation conditions (e.g., $\beta \leq 1$), an initial allocation serves as a local welfare optimum. Otherwise, an internal point may be the welfare optimum, and less unlicensed bandwidth is required. Certain input values may also lead to variations of our model structure. For example, where wide disparities in the congestion sensitivities for service applications exist—meaning consumers are either highly congestion elastic or the opposite—initial strategic equilibria may be skewed toward smaller unlicensed bandwidth allocations. Additionally, where the relationship between service valuations and congestion elasticities are consistent across consumers (e.g., $\frac{w_h}{\lambda_h} \approx \frac{w_l}{\lambda_l}$), it may be advantageous for an *SP* to accelerate its service differentiation, imposing significant congestion externalities and requires excessive unlicensed allocations to recoup all welfare losses.

7 CONCLUSION

We seek to analyze an under-addressed component of the policy discourse for spectrum allocation. Understanding how congestion can impact welfare outcomes is central to determining the most efficient administration of the airwaves. Recent technological developments also enable the aggregation of unlicensed spectrum bands with the exclusively licensed bands held by carriers. As a result, traditional property rights regimes become interdependent, and new economic considerations arise. A unified network improves flexibility; however, it also invites strategic behavior.

We address this emerging environment and how it coincides with the potential for suboptimal resource provisioning and path dependence. Our discussion references the dynamics of congestion pricing, selfish routing, and market-style mechanisms for impure public goods.

We present a simultaneous game where agents (e.g., *SP*s) maximize their payoff by setting prices and provisioning service for a heterogeneous consumer population. Each *SP* has access to an exclusive spectrum license and a second, nonexclusive spectrum commons. As shown, an *SP* may arrive at a pure strategy where it differentiates service based on congestion, effectively leveraging the excludability of its licensed resources. The strategy maximizes individual payoffs but can lead to suboptimal social outcomes through a confluence of price differentiation and negative congestion externalities. These outcomes are reminiscent of Braess's Paradox, in which adding unlicensed capacity—and therefore increasing available resources for market entry and competition—may degrade social and consumer welfare.

Our model demonstrates an *SP* may more aggressively engage in service differentiation, and therefore refrain from provisioning resources for low-priority or congestion inelastic consumer traffic, for larger unlicensed spectrum allocations. Overall, the sequence of pure strategies corresponds to multiple instances of welfare gain and welfare loss, as well as a discontinuity that is coincident with third-degree price discrimination. As shown by example, the impacts on welfare are significant. We also present several output measures on pricing, consumer service populations, revenue, and consumer surplus. Together, these dynamics evidence an emerging dimension to market competition in wireless services where service quality is an increasingly emphasized complement to price and resource availability. By taking these dynamics into account, a more informed calculus can emerge for spectrum policymaking, which we demonstrate through recent allocation decisions in the United States.

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