Vanderbilt University Law School
Law and Economics

Mergers in Auctions with an Incumbent Advantage

Luke Froeb
Vanderbilt University – Owen Graduate School of Management

Mikhael Shor
Vanderbilt University – Owen Graduate School of Management

Steven Tschantz
Vanderbilt University – Department of Mathematics

Mergers in Auctions with an Incumbent Advantage[*]

Luke Froeb[a]   Mikhael Shor[a]   Steven Tschantz[b]

[a]Owen Graduate School of Management
[b]Department of Mathematics
Vanderbilt University

luke.froeb@owen.vanderbilt.edu
mike.shor@owen.vanderbilt.edu
tschantz@math.vanderbilt.edu

─────────────────────────────

PRELIMINARY: COMMENTS WELCOME

─────────────────────────────

September 2008

Abstract

When the winner of one auction gains a cost advantage in the next, bids reflect not only the value of winning the auction, but also the value of gaining an incumbent advantage in future auctions. If a larger firm's advantage derives from a cost or product advantage, it has a greater chance of holding onto incumbency status which, in turn, increases the value it places on gaining incumbency. As a consequence, larger firms bid more aggressively than their smaller rivals, where "size" is measured by the probability of winning. In this environment, mergers eliminate competition among the merged firms but they also change bidding behavior by both merging and non-merging firms. Computational experiments suggest that the scope for pro-competitive mergers is much wider than in auctions without an incumbent advantage. In particular, mergers among smaller firms are likely to be pro-competitive because they tend to create better losers, i.e., firms who bid more aggressively but still lose a large part of the time.

Keywords: dynamic game, auction, incumbent advantage, switching cost, merger, antitrust

JEL Codes: D44 (auctions), C72 (Dynamic Games), L41 (Horizontal Anticompetitive Practices)

1   Introduction

In repeated procurement auctions, the winner of one auction may gain a cost advantage in the next. Such an "incumbent advantage" may be due to learning or sunk-cost expenditures and is present in industries ranging from road building (Silva, Dunne and Kosmopoulou 2003) and business insurance (See Opticare *v.* Marsh & McLennan, 2004) to mainframe computers (Greenstein 1993), IT projects, and mobile telephony (Klemperer 2007). We analyze competition in this environment in order to better understand the effects of mergers.

A closely related literature studies Bertrand and Cournot competition in the presence of customer switching costs. In these models, larger firms price less aggressively because their large base of locked-in consumers makes demand less elastic (Klemperer 1987, 1995, Farrell and Shapiro 1988, Farrell and Klemperer 2007). As a consequence, mergers that eliminate small, more aggressive, firms are considered problematic (Lofaro and Ridyard 2003). The UK Competition Commission appealed to this intuition when challenging a merger involving a bank with only a five percent market share:

> "[I]n markets with switching costs, firms with low market share tended to grow (or sow) their share by competing aggressively . . ., while those with high market share tended to exploit (or harvest) theirs by preserving or increasing margins on the existing customer base. The merger . . . would replace a firm in sowing phase with one in harvesting phase, to the detriment of consumers and competition" (Lofaro and Ridyard 2003, p. 2).

In an auction setting, we find the opposite. If a larger firm's advantage derives from lower costs or superior products, it has a greater chance of holding onto incumbency status which increases the value it places on gaining incumbency. As a consequence, larger firms bid more aggressively than their smaller rivals, where "size" is measured by the probability of winning. A merged entity, which obtains the lower of its constituent costs (or the maximum of its constituent values), typically bids more aggressively than it did pre-merger which can offset the loss of bidding competition between the merging firms. Heuristically, the losing bidders determine the price, and if the merger creates more aggressive losing bidders, on average, price can decrease.

To isolate the the effect of incumbent advantage, we consider mergers or bidding coalitions in the simplest of auction formats: a second-price, private-value auction. Not only does the second-price format revenue-dominate the first-price auction in settings with incumbent advantage (Jeitschko and Wolfstetter 2002, Leufkens and Peeters 2007), in settings without an incumbent advantage, the dominant strategy equilibrium implies that merger effects arise only when the merged coalition contains the two lowest-cost firms. The loss of competition between the merging bidders increases price from the second- to the third-lowest cost but does not otherwise affect bidding behavior or winning probabilities. Absent efficiency gains, these mergers always raise price (e.g., Froeb, Tschantz and Crooke 2001).

This simple merger characterization has been used by antitrust enforcement agencies to quantify the price effects of mergers between hospitals, mining equipment companies, and defense contractors (Baker 1997). More importantly for our purposes, this merger model gives us a benchmark that allows us to isolate the effect of incumbent advantage on mergers and to answer the policy question that motivates our interest in the topic: how should the antitrust authorities analyze mergers among bidders in auctions with an incumbent advantage?

With an incumbent advantage, the competitive equilibrium in a repeated second-price auction changes from a simple dominant strategy equilibrium to a Markov-perfect equilibrium without dominant strategies. Bidders face a strategic dependency that is akin in complexity to that between bidders in a first-price auction. In these auctions, analytical closed-form bidding functions are available only in highly stylized cases, typically with only two types of bidders (e.g., Plum 1992, Lebrun 1999, Maskin and Riley 2000, Arozamena and Cantillon 2004, Kaplan and Zamir 2007). Several authors have used these models to study the effects of incumbent advantage. Jeitschko and Wolfstetter (2002) and Leufkens and Peeters (2007) use the results of Plum (1992) to solve for the equilibrium in a two-period two-bidder model with incumbent advantage, finding that bidding in the first period is more aggressive than in the second, accounting for the value of incumbent advantage. Tang Sørensen (2006) and von der Fehr and Riis (2000) consider a two-period n-bidder second-price auction in which bidders do not learn their values (costs) in the second period until after the first auction. Prices may rise or fall from the first to the second period, and are not necessarily monotonic in the number of bidders.

We depart from these models in two ways. First, we model an infinite-horizon repeated setting where bids account for the value of future incumbency in each period. Second, we allow for a richer characterization of asymmetry than can be analyzed analytically.[1] This is necessary because analysis of mergers requires that we model two types of asymmetry: the asymmetry created by merger, since even a symmetric industry will become asymmetry post-merger, and the asymmetry that arise from incumbency, where the incumbent firm draws from a more favorable cost or value distribution. This requisite modeling complexity precludes analytic closed-form solutions. Instead, we gain insight into the model predictions numerically, through a series of computational experiments.

The papers surveyed above all assume that the auctioneer uses a simple (first or second price) auction in each period and does not adjust to the asymmetry created by incumbent advantage. Though these simple auctions are not optimal, we follow this approach both because this allows more direct comparisons to past studies and because it often closely approximates real behavior. In government procurement, for example, it is not uncommon (and often required by law) for auctioneers to ignore the asymmetries that arise from incumbency (e.g., **?**)(e.g., Jofre-Bonet and Pesendorfer and Porter and Zona in highway paving contracts, European Union directive for incumbent-irrelevant low-cost competition, etc.).[2] Despite the costs of doing so (Brannman and Froeb 2000), governments generally eschew allowing bidder-specific asymmetric auctions due to the heavy informational requirements and fear of political opportunism (Appelman, Gorter, Lijesen, Onderstal and Venniker 2003).

Our results suggest that the scope for pro-competitive mergers is much wider than in auctions without an incumbent advantage. In particular, mergers among sufficiently small firms are likely to be pro-competitive. As the incumbent's advantage increases, the set of pro-competitive mergers also grows.

In what follows, we consider bidding in markets with an incumbent advantage, which naturally gives rise to a dynamic model of competition. We contrast this in a series of computational experiments to a static model of bidding competition without an incumbent advantage. We conclude with a policy discussion.

---

[1]We do offer an analytic solution to the *ex ante* symmetric case which can be viewed as the infinite horizon version of Tang Sørensen's (2006) model.

[2]See, for example, Porter and Zona (1993) and Jofre-Bonet and Pesendorfer (2000). The European Union's public procurement regulations do not permit unequal treatment of incumbents (EU 2004). McAfee and McMillan (1988) provide some examples of companies showing preference for non-incumbents, though Elmaghraby (2007) discovers that this is a rare occurrence.

3

2 Bidding with an incumbent advantage

2.1 Model

Consider an infinite-period dynamic game between n firms, indexed by $j = 1, \ldots, n$, competing in a second-price procurement auction in each of $t = 1, 2, \ldots$ periods. We define $i^*(t)$ as the index of the incumbent bidder in period t, so that $i^*(t) = j$ if bidder j won the auction in period $t-1$ (with $i^*(1) = 0$). Every period, each firm receives an independent (though not necessarily identically distributed) cost draw, $C_j(t)$, from a time-invariant distribution $F_j(C)$ with density $f_j(C)$ and support $\mathbb{C}_j$. Costs are given by

$$c_j(t) = \begin{cases} C_j(t) & j \neq i^*(t) \\ C_j(t) - c_{inc} & j = i^*(t) \end{cases}$$

We use C to denote the elementary cost draws, and c to denote realized costs, with $c = C$ for all but the incumbent bidder, which receives a cost advantage of c_{inc}.

Denote by $v_j(i, t)$ firm $j's$ net present value of expected future profits at time t when i is the incumbent. Let r denote the interest rate and $p_j(i, t)$ denote firm $j's$ probability of winning the auction at time t when i is the incumbent. Define

$$\Phi_j(i^*(t), t) = \left(v_j(j, t+1) - \frac{\sum_{k \neq j} p_k(i^*(t), t) v_j(k, t+1)}{\sum_{k \neq j} p_k(i^*(t), t)} \right) \Big/ (1+r) \tag{1}$$

as the difference in future profits for bidder j between winning and losing today's auction. This is the additional value, above that implied by the profits in the current period, that a bidder places on winning and is analogous to the *intrinsic valuation* in auctions with externalities.[3] In these models, losing bidders suffer a negative externality which may depend on the identity of the winning bidder. In our case, this externality is reflected in the more preferable cost distribution for the incumbent in the subsequent period. Note that in an auction without an incumbent advantage, $v_j(i, t) = v_j(k, t) \, \forall i, j, k$ and $\Phi_j(i^*(t), t) = 0$.

We consider stationary strategies that depend only on the state variables, namely the identity of the incumbent and a firm's realized cost. This allows us to drop the dependence of $v_j(i^*)$, $p_j(i^*)$, and $\Phi_j(i^*)$ on time. A strategy, $b_j : \Re \times \{1, \ldots, n\} \rightarrow \Re$, maps costs into bids of player j for each possible incumbent. The Markov Perfect equilibrium bid in a second-price auction is given by:

$$b_j(c_j, i^*) = c_j - \Phi_j(i^*) \tag{2}$$

where Φ_j is the amount by which each bidder "shades" his realized cost to form bids. Note that $\Phi_j(i^*)$ depends on the winning probabilities of each firm which, in turn, depend on the bids. Denoting by $b_{-j}^{min}(i^*)$ the lowest bid of the bidders excluding j, $\{1, \ldots, j-1, j+1, \ldots, n\}$, the value function must satisfy the recursion equation:

$$v_j(i^*) = \left(E[b_{-j}^{min}(i^*) | b_{-j}^{min}(i^*) > b_j(c_j, i^*)] - c_j \right) p_j(i^*) + \sum_{k=1}^{n} p_k(i^*) v_j(k)/(1+r) \tag{3}$$

where the first term is the present period's profit and the second term is next period's value function for each incumbent weighted by the transition probabilities.

---

[3] See Jehiel, Moldovanu and Stacchetti (1996), Jehiel and Moldovanu (1996, 2000), Das Varma (2002, 2003), and Bagchi (2007).

4

2.2 Logit Auction Model

To characterize equilibrium in a parsimonius and tractable way, we use the logit auction model (e.g., Tschantz, Crooke and Froeb 2000, Brannman and Froeb 2000, Froeb, Tschantz and Crooke 2001). Bidder j's cost absent any incumbent advantage follows an extreme-value (Gumbel) distribution,

$$F_j(C) = 1 - e^{-e^{(C-\eta_j)/\mu}} \tag{4}$$

with common variance given by the spread parameter μ, but possibly different location parameters, η_j. These distributions offer two advantages for the analysis of mergers.

First, the distributions offer a natural interpretation for how a merged firm draws its cost. In "power related distributions" (Waehrer and Perry 2003, Froeb, Tschantz and Crooke 2001), we imagine that bidder j receives x_j draws out of a common base distribution, $F(C)$, with his cost equaling the lowest draw. Bidder j's distribution is given by

$$F_j(C) = 1 - (1 - F(C))^{x_j} \tag{5}$$

where $F(C) = 1 - e^{-e^{C/\mu}}$ and $x_j = e^{-\eta_j/\mu}$. A bidder taking a larger number of draws will have lower costs, a higher probability of winning, and will win at a better prices, on average, than those taking fewer draws (Froeb, Tschantz and Crooke 2001). If a merged firm obtains a cost equal to its constituent members' lowest cost draw, then a merger of firms with parameters x_i and x_j has a cost distribution with parameter $x_{merged} = x_i + x_j$.

A second benefit of the Gumbel distribution is that the distribution for the merged firm's costs exhibits the same variance as do the distributions of its constituent members. Therefore, we can concentrate on the effects of mergers without the confounding effect of variance reduction.

2.3 Equilibrium

We begin with the symmetric equilibrium both because it is analytically tractable and because it serves as a reference point for the computational experiments that follow. In particular, we examine mergers to a symmetric equilibrium (post-merger symmetry) and mergers from a symmetric equilibrium (pre-merger symmetry). The symmetric equilibrium has the characteristic that incumbent advantage does not affect shares; the conditional probability of winning an auction in which one is the incumbent varies, but the long-term market shares do not. An increase in incumbent advantage increases the gains from winning the auction which induces more aggressive bidding on the part of all bidders.

Bidders receive *i.i.d.* draws from the distribution $f_j(C) \equiv f(C)$ which implies $v_j(j) \equiv v_{inc}$ and $v_k(j) \equiv v_{non}, \forall j, k, \ j \neq k$. Similarly, denote by p_{inc} and $p_{non} = (1 - p_{inc})/(n-1)$ the probability of an incumbent and non-incumbent winning the auction. Equation (1) reduces to

$$\Phi_j(k) \equiv \Phi = (v_{inc} - v_{non})/(1 + r) \tag{6}$$

for all j and k.

In the absence of an incumbent advantage, bidder j's surplus from winning the current auction when he has cost c_j is equal to $E[\min\{c_{-j}\}|\min\{c_{-j}\} > c_j] - c_j$ where c_{-j} is the vector of costs of bidder $j's$

rivals. We denote this surplus by $\Delta_{inc}^{f}(c)$ when the incumbent wins with cost c and by $\Delta_{non}^{f}(c)$ when a non-incumbent wins. These clearly depend on the distribution, f. With an incumbent advantage, the winner additionally pays Φ. This yields the following expressions:

$$v_{non} = (\Delta_{non} - \Phi)p_{non} + \frac{v_{inc}p_{non} + v_{non}(1 - p_{non})}{1 + r} \tag{7}$$

$$v_{inc} = (\Delta_{inc} - \Phi)p_{inc} + \frac{v_{inc}p_{inc} + v_{non}(1 - p_{inc})}{1 + r} \tag{8}$$

The first terms reflect the profits from today's auction and the second terms reflects the discounted profit of future auctions which depends on the current auction's outcome. Solving for v_{inc} and v_{non} and substituting into (6), we obtain

$$\Phi = \frac{\Delta_{inc}p_{inc} - \Delta_{non}p_{non}}{1 + r} \tag{9}$$

This expression indicates that each bidder bids below his costs ("shades") by the same amount, whether or not he is the incumbent, as Φ does not depend on incumbency status. There is no "bargain-then-ripoff" bidding as in standard models of switching costs, (e.g., Farrell and Klemperer 2007). There are several reasons for this difference. First, two-period models of competition with switching costs offer no trade-off for firms in the second period; high prices to existing customers in the final period are not fully balanced against the competition for new customers. Second, in repeated settings, switching costs provide some inertia, as a customer, once gained, may remain a customer for many periods. Although there is a direct analogy between "installed base" in switching cost models and "incumbency status" in our auction model, the winner-take-all nature of auctions suggests that incumbency status in auctions is much more fleeting. If one were to exploit or, to use the terminology of the UK Competition Commission, "harvest" incumbency status by bidding too high, this would lead to a greater chance of loss, and thus a higher likelihood that the current incumbent would be disadvantaged in the following period. Lastly, incumbency does not reduce bidding aggressiveness in a second-price auction the same way that it raises prices in price-setting models since bids determine only who wins the auction, not the price the winner pays.

The expression for Φ above provides only an implicit solution for the bidding function, since p_{inc} is determined by the bids, and the bids depend on Φ which is a function of p_{inc}. When bidders draw *iid* costs from the Gumbel distribution, it is possible to derive explicit expressions for winning probabilities,

$$p_{inc} = \Pr\{b_j(c_j, j) < b_i(c_i, j) \; \forall i \neq j\} = \Pr\{C_j < C_i + c_{inc} \; \forall i \neq j\} \tag{10}$$

$$= \frac{e^{c_{inc}}}{n - 1 + e^{c_{inc}}} \tag{11}$$

$$p_{non} = (1 - p_{inc})/(n - 1) \tag{12}$$

$$= \frac{1}{n - 1 + e^{c_{inc}}} \tag{13}$$

6

as well as equilibrium bids and expected profit,

$$b(c) = c - \Phi = c - \log\left(1 + \frac{e^{c_{inc}} - 1}{n - 1}\right) \Big/ (1 + r) \tag{14}$$

$$\Delta_k p_k = -\mu \log(1 - p_k), \quad k \in \{inc, non\} \tag{15}$$

Bids are decreasing in the incumbent advantage, c_{inc}, and equal c when the incumbent advantage is zero. That larger incumbent advantage encourages more aggressive bidding is not surprising. More interesting is the ratio of the amount of bid-shading to the incumbent advantage, $\frac{\Phi}{c_{inc}}$. This ratio equals $\frac{1}{1+r}$ when $n = 2$ suggesting that firms incorporate the entire present value of next period's cost advantage into their bid. The ratio is increasing in c_{inc}, since incumbency is easier to preserve once obtained, and decreasing in n since incumbency becomes more difficult to maintain (and therefore worth less to obtain) with more competitors.

An incumbent advantage reduces the equilibrium price not only through the direct effect of the incumbent's lower average cost but also indirectly through the more aggressive bidding. Thus, higher c_{inc} benefits the auctioneer, who plays the role of the consumer in this model.

As mentioned in the introduction, a symmetric setting is inappropriate for our application due to the two types of symmetry our model must accommodate, asymmetry between firms and and asymmetry created by incumbency status. The asymmetric model eludes closed-form expressions, so we compute a numerical solution to the asymmetric equilibrium defined by the following system of simultaneous equations for $\Phi_j(i)$ for all i, j pairs.

Proposition 1. *The equilibrium of the corresponding second-price incumbent-advantage logit auction is given by a system of a system of n^2 equations in $\Phi_j(i)$, for $i, j \in \{1, \ldots, n\}$.*

$$b_j(c_j, i) = c_j - \Phi_j(i) \tag{16}$$

where

$$\Phi_j(i) = \frac{h_j(j) - \Phi_j(j) - \sum_{k=1}^{n} p_k(i)\Big(h_j(k) - \Phi_j(k)\Big)}{(1 + r)(1 - p_j(i))} \tag{17}$$

$$\Phi_j(i) = \left(h_j(j) - \Phi_j(j) - \frac{\sum_{k \neq j} p_k(i)\Big(h_j(k) - \Phi_j(k)\Big)}{1 - p_j(i)}\right) \Big/ (1 + r) \tag{18}$$

$$h_j(k) = -\log(1 - p_j(k)) \tag{19}$$

$$p_j(i) = \frac{\alpha_j x_i}{\sum_{k=1}^{n} \alpha_k x_k} \qquad where\ \alpha_k = \begin{cases} e^{\Phi_k(i)} & k \neq i \\ e^{\Phi_k(i) + c_{inc}} & k = i \end{cases} \tag{20}$$

Unlike the symmetric equilibrium, the level of bid shading, $\Phi_j(i)$, depends on incumbency status, as well as the degree of incumbent advantage, c_{inc}. However, this system of equations is transcendental, and does not have analytic solutions. Numerical solutions are obtained in *Mathematica* using Brent's (2002) extension to the secant method.

3 Computational Experiments

In this section, we present results for two sets of computational experiments, designed to isolate the effect of an incumbent advantage on a merger. We hold the underlying cost distributions of the bidders fixed and define $\mathbf{x} = (x_1, x_2, x_3, \ldots, x_n)$ as a vector of cost parameters, $x_j > 0$. A merger among firms 1 and 2 in an n-firm industry given by $\mathbf{x} = (x_1, x_2, x_3, \ldots, x_n)$ creates an industry with $n - 1$ bidders with cost parameters $(x_1 + x_2, x_3, \ldots, x_n)$. We set the spread parameter, $\mu = 1$, and for ease of interpretation, normalize $\sum x_j = 1$, so that x_j is precisely firm j's probability of winning without incumbent advantage.
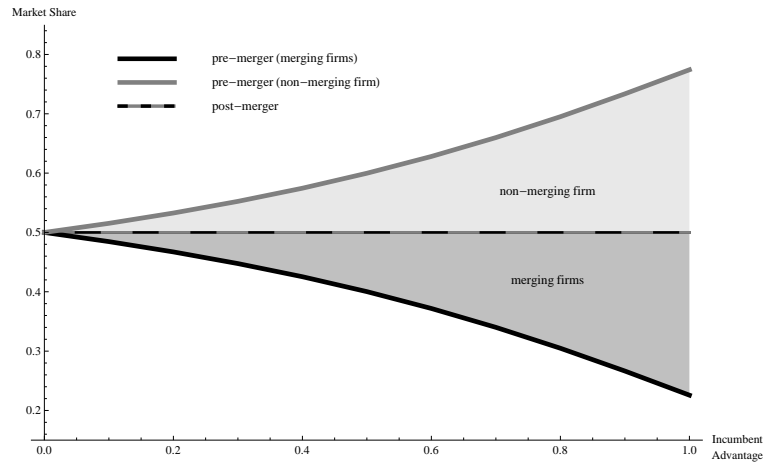
3.1 Mergers to symmetry

In this section, we compute the effects of moving from a pre-merger industry of $n + 1$ firms with cost parameters $(1/(2n), 1/(2n), 1/n, \ldots, 1/n)$ to a post-merger industry of n firms with cost distribution of $(1/n, 1/n \ldots, 1/n)$. Thus, pre-merger, firms one and two have less favorable cost distributions, while post-merger the industry is symmetric. In the computational experiments that follow, we reach three conclusions about the effects of incumbent advantage on mergers:

- merger increases the combined share of the merging firms,

- merger increases bidding aggressiveness by the merging firms, and

- mergers are always profitable and often reduce price.

To show this, we begin with a three-to-two merger that moves an industry from $\mathbf{x} = (1/4, 1/4, 1/2)$ to a post-merger industry characterized by $\mathbf{x} = (1/2, 1/2)$. Figure 1 illustrates the merger effects as the incumbent advantage c_{inc} varies from zero to one. In the top panel, we illustrate the effect of the merger on market shares. Pre-merger (solid line), the merging firms' total market share (black line) is the mirror image of the non-merging firm's share (gray line). As incumbent advantage increases, the market share of the larger firm also increases. Post-merger (dashed line), the merging and non-merging shares are equal at 50%. When incumbent advantage equals zero, the sum of the shares of the pre-merger firms equals the share of the post-merger firm, so mergers have no effect on shares. This is because the non-merging firm's probability of having the lowest cost draw does not change with the merger of its rivals. With incumbent advantage, since the merger always results in symmetric market shares, the merger has the effect of increasing the share of the merging firm, while decreasing the share of the non-merging firm.

In the middle panel, we plot the average level of bid shading below one's costs of the merging and non-merging bidders; the average of $\Phi_j(i)$ weighed by each firm's market share. The merging firm bids much more aggressively following the merger, while bidding behavior of the non-merging firm remains largely unchanged. The more aggressive bidding by the merged firm causes its share to increase in the top panel.

In the bottom panel, we consider the incentives to merge and the merger's effect on price. For low levels of incumbent advantage, the merger is anticompetitive as it causes an increase in price. For sufficiently large incumbent advantage, the more aggressive bidding by the merging firm attenuates the merger effect and can cause price to decline. In addition, we see that the mergers to symmetry are profitable over the entire range of incumbent advantage, even when the merger causes expected price to fall.

(a) Shares
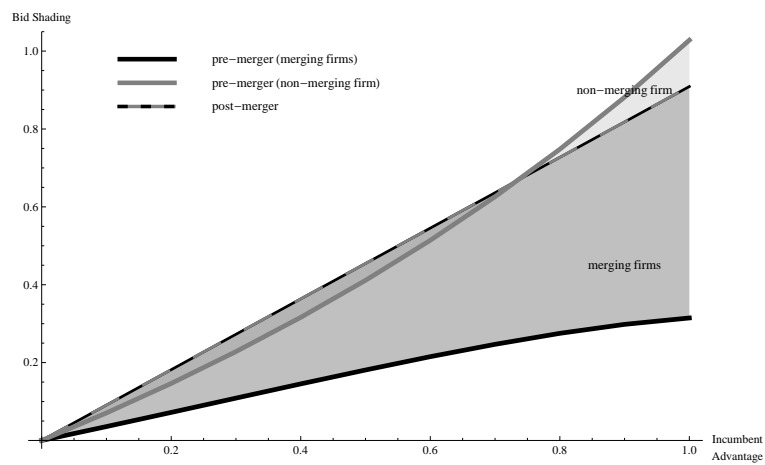
(b) Bid Shading
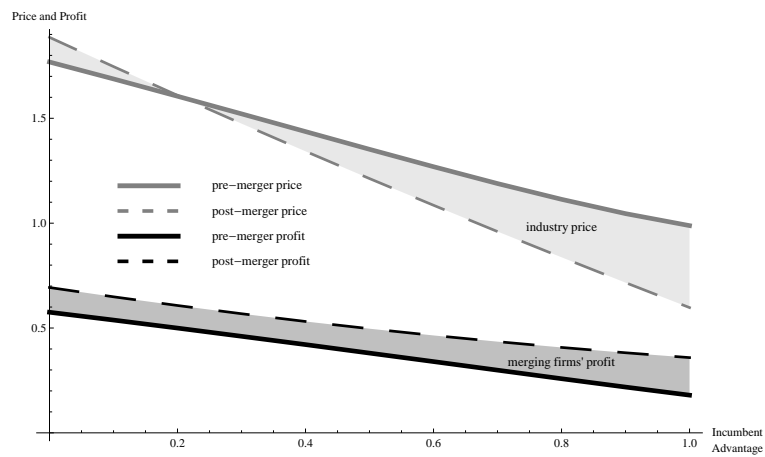
(c) Prices and Profit

Figure 1: Merger to Symmetry: from a pre-merger cost distribution of (1/4, 1/4, 1/2) to a post-merger distribution of (1/2, 1/2). In each of the panels, the solid line represents the pre-merger equilibrium, and the dotted line, the post-merger equilibrium. In top panel of this figure, the merging firm *gains* market share; in the middle panel, the merging firm bids much more aggressively post-merger, explaining its post-merger gain in share; and in the bottom panel, mergers are always profitable, and can reduce expected post-merger price.
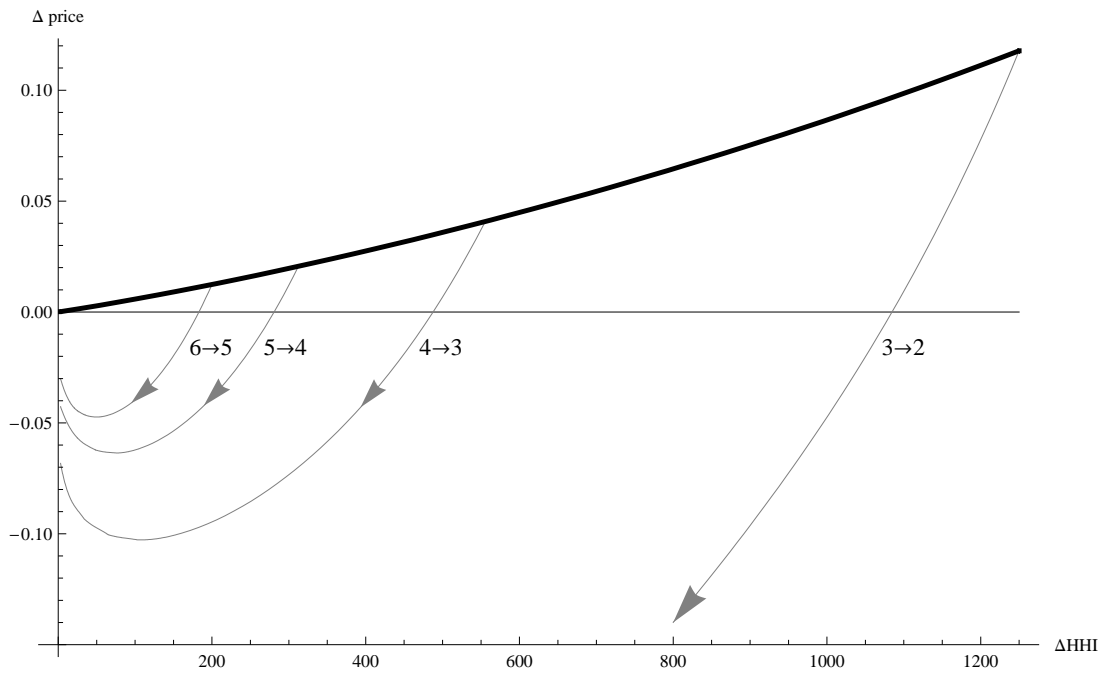
9

Figure 2: Mergers to Symmetry: $\Delta Price$ vs. ΔHHI. The thick line indicates the merger effect of two symmetric firms in the absence of incumbent advantage. Thinner lines indicate the merger effects when the post-merger industry is symmetric. An increase in incumbent advantage (in direction of arrow) makes the pre-merger asymmetric equilibrium less competitive, so a symmetry-restoring merger has a bigger benefit. For sufficiently large levels of incumbent advantage, a merger actually benefits consumers.

For all but very small incumbent advantage, mergers to symmetry in the underlying cost distribution are pro-competitive. The challenge for antitrust practice, however, is that observed firm shares do not directly translate into unobserved parameters of cost distributions, as the link is conflated with incumbent advantage. To fix ideas, consider an industry with three firms, two of which each have 10% market share. Without incumbent advantage, a firm with 10% market share in a second-price auction has precisely a 10% chance of having the lowest cost. With incumbent advantage, asymmetries in the cost distribution are reflected in even larger asymmetries in observed shares. Firms with 10% market share might have a 25% chance of having the lowest cost but the more aggressive bidding by their larger rival reduces their observed shares. If these two smaller firms merge, the resulting industry would be symmetric, and their shares would increase from a combined 20% to 50%. A simple analysis that merely adds the shares of the two smaller firms to determine the post-merger share will, of course, conclude that the post-merger HHI is higher than pre-merger. This is only part of the story. When we observe the merging firms' shares at 10% each, the symmetry-inducing merger actually leads to a decrease in the HHI and a decrease in prices post-merger. Firm shares, without knowledge of the incumbent advantage, do not deterministically characterize the impact of a merger.

In Figure 2, we illustrate the indeterminacy of price effects of mergers. In the absence of incumbent advantage, the thick line illustrates the price effects of a merger between two identically-sized firms as a function of the change in HHI, $\Delta HHI = 2x_i^2$. In the presence of incumbent advantage, observed market shares do not accurately reflect the underlying cost distributions of firms. The thinner lines plot the change in price resulting from mergers to cost symmetry against the change in ΔHHI computed from pre-merger shares as we increase the incumbent cost advantage c_{inc} from 0 to 1. The arrow denotes

10

the direction of increasing incumbent advantage. Thus, the thick line shows the predicted merger effects using the static model of auctions without an incumbent advantage and the thin lines show the best case scenario, where the cost advantage is such that the firms would be symmetric post-merger. In principle, for given shares (or predicted HHI change), any price effect between the thin and thick line is possible for some level of incumbent advantage.

3.2 Mergers from symmetry

In this section, we consider the change from n symmetric firms with pre-merger cost parameters given by $(1/n, 1/n, 1/n, \ldots, 1/n)$ to a post-merger cost distribution of $(2/n, 1/n \ldots, 1/n)$ caused by a merger of firms 1 and 2. Thus, all firms are identical pre-merger, but the merged firm has the most favorable cost distribution following the merger. In the computational experiments that follow, we reach three conclusions about the effects incumbent advantage on merger. Note that the first two conclusions are the same as for mergers from symmetry, but the third is more qualified.

- merger increases the combined share of the merging firms,

- merger increases bidding aggressiveness by the merging firms, and

- mergers are always profitable and sometimes reduce price.

To show this, we begin with a three-to-two merger that moves an industry with $\mathbf{x} = (1/3, 1/3, 1/3)$ to a post-merger industry with $\mathbf{x} = (2/3, 1/3)$. In the top panel of Figure 3, the pre-merger industry is symmetric, so the merging firms have combined twice the market share of the non-merging firm and these shares do not vary with incumbent advantage. Post-merger, however, the merged firm's market share (black) grows with incumbent advantage. As in the previous section, increasing the incumbent advantage exacerbates cost asymmetries, increasing the share of the lower-cost, merged firm.

The middle panel shows that the merging and non-merging firms shade their bids equally in the pre-merger symmetric world. It is only in the post-merger world, as incumbent advantage grows, that the lower-cost merging firm bids more aggressively, accounting for the share increase in the top panel. The non-merging firm also bids slightly more aggressively post-merger for low levels of incumbent advantage, so that when the incumbent advantage is below 0.4, the merger causes both firms to bid more aggressively. For an incumbent advantage above 0.4, the non-merging firm begins to bid less aggressively, effectively ceding share to the lower-cost merged rival.

The bottom panel of Figure 2 reflects both the incentive to merge and the anti-competitive effects of mergers. This three-to-two merger from symmetry is profitable over the entire range of incumbent advantage and always causes price to increase. However, the more aggressive bidding by both bidders for low levels of incumbent advantage suggests that the price effects of a merger are first decreasing than increasing in incumbent advantage (lightly shaded area in panel c). Thus, in a three-to-two merger from symmetry, static models that ignore incumbent advantage will accurately predict the direction of price movements, but may overestimate or underestimate their magnitude. This error is even more pronounced in industries with more than three firms.
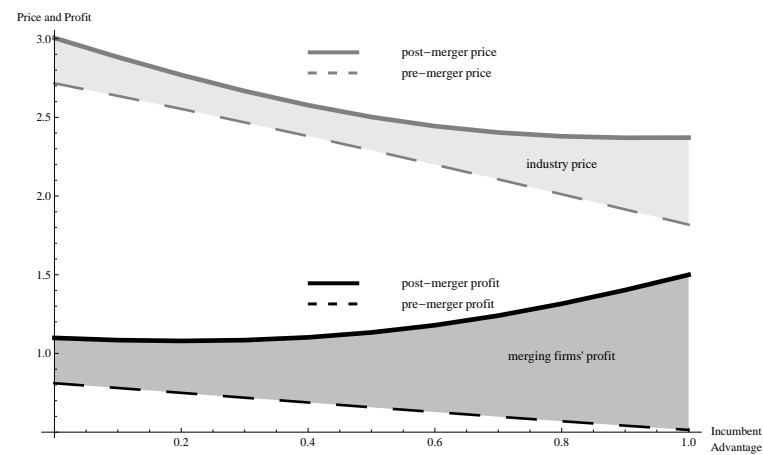
The net effect of mergers on price for industries with varying numbers of symmetric firms is illustrated in Figure 4 which plots the change in price resulting from the merger against the change in actual ΔHHI

11
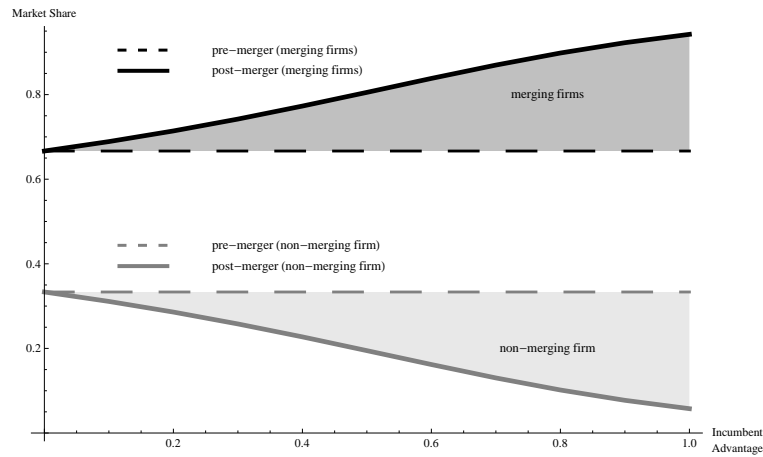
(a) Shares
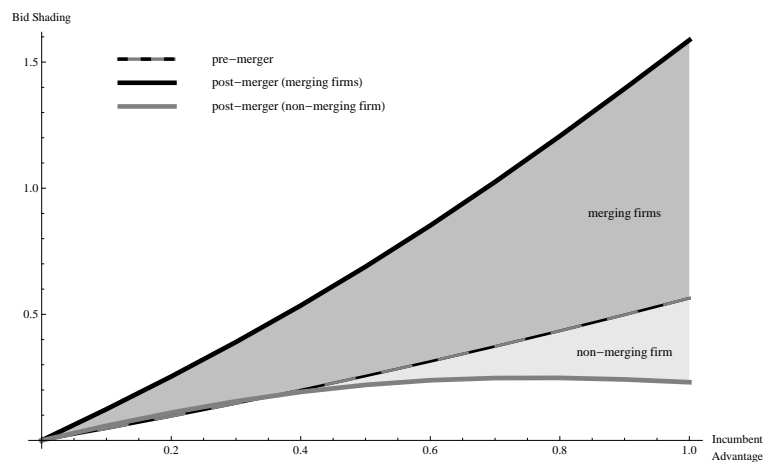
(b) Bid Shading

(c) Prices and Profit

Figure 3: Merger From Symmetry: from a pre-merger cost distribution of (1/3, 1/3, 1/3) to a post-merger cost distribution of (2/3, 1/3). In top panel of this figure, the merging firm *gains* market share; in the middle panel, the merging firm bids much more aggressively post-merger while the non-merging firm bids less aggressively, explaining its share increase; and in the bottom panel, mergers are always profitable, and post-merger price always increases.
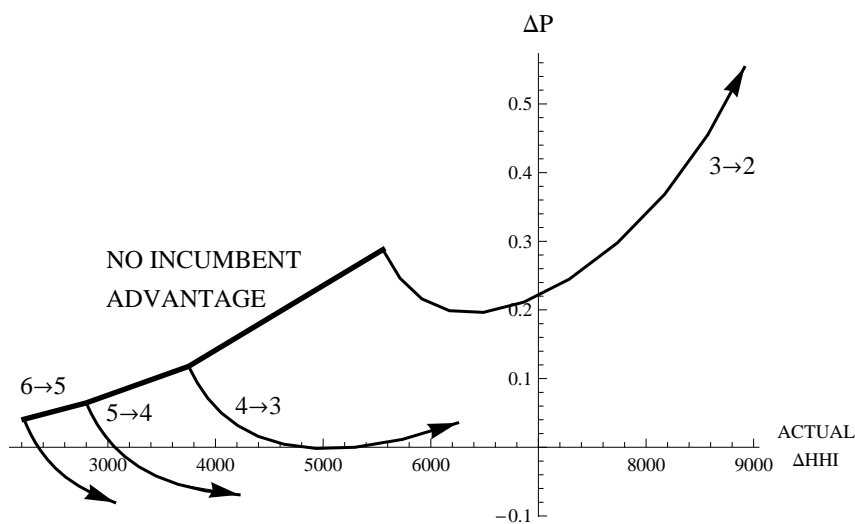
12

Figure 4: Mergers From Symmetry: $\Delta Price$ vs. ΔHHI. This graph presents a parametric plot of the change in price on the change in concentration as the incumbent advantage increases. An increase in incumbent advantage (in direction of arrow) makes the merged firm bid more aggressively and win more frequently. This initially causes price to *decrease* as concentration increases.

as we increase the incumbent cost advantage c_{inc} from 0 to 1. The arrow denotes the direction of increasing incumbent advantage. Unlike Figure 2, we plot *actual* ΔHHI rather than ΔHHI computed from pre-merger shares. The reason for this is that for mergers from symmetry, ΔHHI computed from pre-merger shares would not change as we vary the incumbent advantage.

The striking feature of the graph is that initially all four lines slope downward. Raising the incumbent advantage attenuates the anticompetitive merger effect for small incumbent advantages. For six-to-five and five-to-four mergers, the merger effect becomes negative as the merged firm becomes a better loser. It bids more aggressively, but due to the size of the industry, still does not win too frequently. Thus, it becomes a frequent price setter in a second-price auction where the best losing bidder determines the price. For the three-to-two and four-to-three mergers, the merged firm begins winning too much, so its aggressive bidding serves to reduce price too infrequently.

3.3 Non-symmetric mergers

In the preceding subsections, we considered the polar cases of industries that are symmetric either before or after the merger. In both cases, we assumed that merging firms were of equal size. We next examine how mergers of two firms of arbitrary size impact the resulting price. Figure 5 shows the price effects of a merger between two firms for different market sizes and levels of the incumbent advantage. In mergers from three to two firms (top two panels), mergers among small firms (lower left of each panel) are pro-competitive even under fairly small levels of incumbent advantage. In a six-firm industry, mergers among small firms are unlikely to be pro-competitive when the incumbent advantage is small. Several insights may be gleaned from this figure.

First, the thick isoprice line (which indicates no merger price effect) is nearly linear, suggesting that the sum of merging firms' cost parameters may be a better indicator of merger effects than the sum of the products of their shares, as in the HHI.
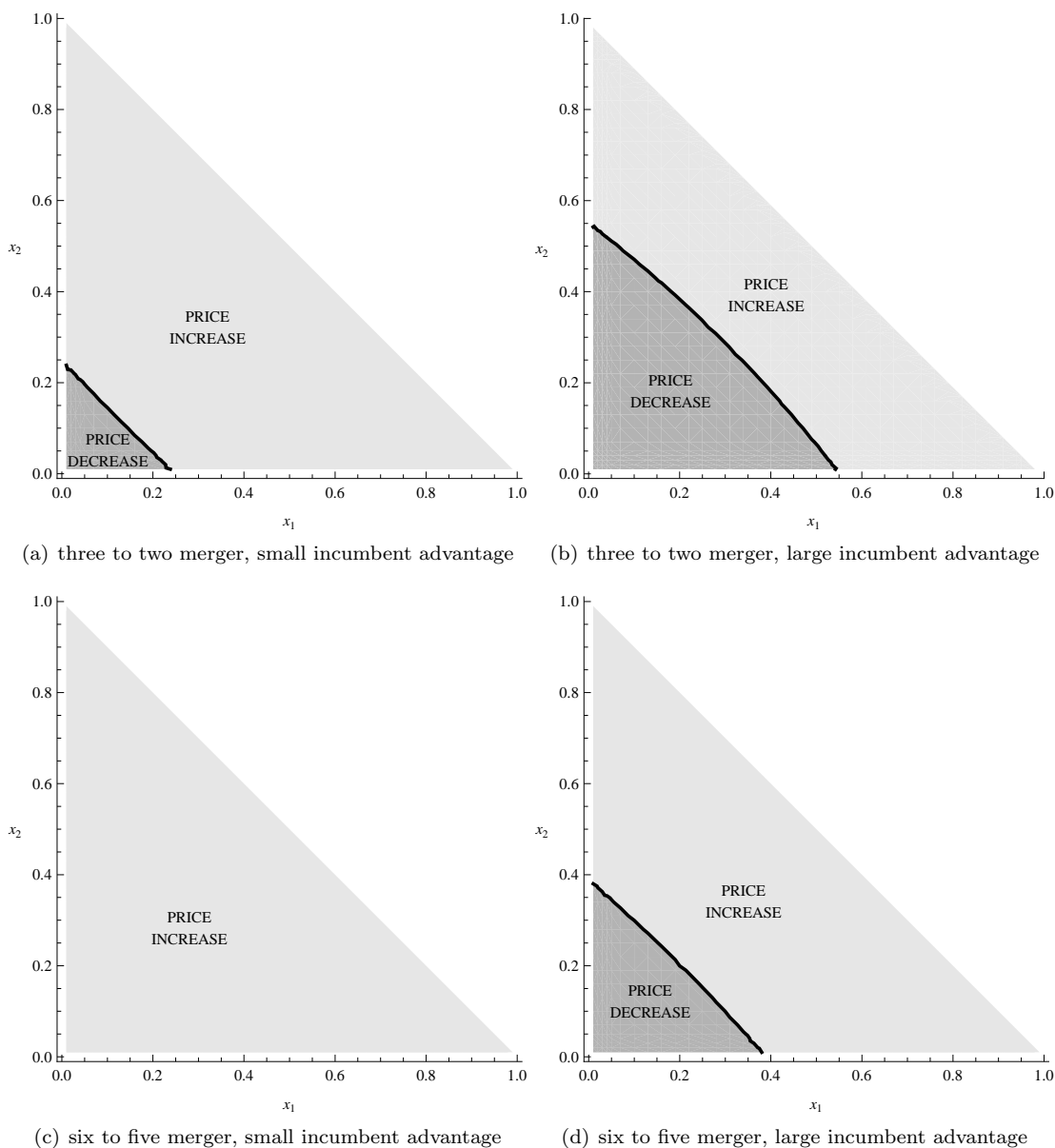
Figure 5: Incumbent advantage increases the scope for pro-competitive mergers. Panels show price effects of a merger among two firms with cost parameters x_1 and x_2 under small ($c_{inc} = 0.05$) and big ($c_{inc} = .5$) incumbent advantage. Lightly shaded region shows an (anticompetitive) increase in price, while darker region illustrates prices declining as a result of merger. With three firms, non-merging firm has cost parameter $1 - x_1 - x_2$; with six firms, each non-merging firm has cost parameter $(1 - x_1 - x_2)/4$. Prices are more likely to decline as a result of merger when merging firms are small, the market has few firms, and incumbent advantage is large.

14

Second, a merger of two firms with given cost parameters is more likely to generate pro-competitive effects in markets with few competitors than in markets with many competitors. This means that the order in which mergers occur determine whether they are anticompetitive or not. Consider for example a six-firm industry with an incumbent advantage of $c_{inc} = .5$ and cost parameters of $x_1 = x_2 = 0.28$ and $x_3 = x_4 = x_5 = x_6 = 0.11$. A merger between firms 1 and 2 would increase price (panel d). However, if the four smaller firms first merge, then a merger between firms 1 and 2 would actually be pro-competitive (panel b). Contrary to the traditional thinking that a consolidating industry observes ever growing anticompetitive effects with each subsequent merger, it is the early mergers in this example that harm the industry, and the latter merger that improves competition.

Third, the above example also demonstrates that even some mergers that create a dominant firm may be pro-competitive. The three-to-two merger of firms 1 and 2 creates a firm with cost parameter 0.56. However, the post-merger environment creates two sufficiently similar competitors; pre-merger, these firms were too small to compete effectively against their larger, often incumbent-advantaged, rival.

4   Conclusion

Farrell and Klemperer (2007) introduce their chapter on switching costs in the *Handbook of Industrial Organization* by warning that the analysis of mergers is different in the presence of switching costs. In an auction instead of a price-setting environment and with an incumbent advantage instead of switching costs, we reach the same conclusion. We find that competitive behavior in repeated auctions with an incumbent advantage is characterized by dynamic concerns analogous to those that characterize competitive behavior in the presence of switching costs. Firms' bids reflect not only the value of winning the current auction but also the value of gaining an incumbent advantage in future auctions. Bidding differs from bidding in a static framework, i.e., without an incumbent advantage, and the welfare effects of mergers depend on the degree of incumbent advantage.

However, rather than justifying "increased antitrust scrutiny" (p. 2005), we find that the competition in this environment gives rise to a greater scope for pro-competitive mergers. In particular, when a merger creates a better losing bidder, one that bids more aggressively but still loses a great deal of the time, mergers are likely to reduce price. For a large enough incumbent advantage, mergers to symmetry reduce price because they make the merged firm a better loser. Mergers from symmetry can also reduce price, but if they create a dominant firm, they can raise price. The sum of the cost parameters, representing the probability of winning in an auction without an incumbent advantage, is a better predictor of whether a merger will be anticompetitive than is the change in the HHI.

This conclusion may not be very useful for distinguishing pro- from anti-competitive mergers because the relationship between unobserved costs and observed shares is confounded by the magnitude of incumbent advantage. Determining how to empirically identify the cost parameters from the share parameters would seem like a worthwhile econometric exercise. One approach may exploit observed renewal rates, the probability with which an incumbent wins. If costs are independently drawn in each period, each firm's probability of winning should be constant. Larger incumbent advantages might be reflected in higher likelihoods that firms win as incumbents than as non-incumbents. Without knowing whether a firm's large winning probability is due to increased incumbent advantage or an inherent product or cost advantage, both type I (over-deterrence) and type II (under-deterrence) enforcement errors are possible.

We model incumbent advantage as short-lived, lasting only as long as a firm remains an incumbent

15

and disappearing as soon as a firm loses. This is an accurate description of cases where sunk costs must be incurred with each new contract. If incumbency leads to long-lived advantages, such as learning curves which decay over time, we speculate that similar results will likely obtain. The current model where a current state is defined by the index of the current incumbent would instead require a definition of a state as a vector of each firm's current stock of learning. Further, the value of next period's incumbency would replace a single-period benefit with a discounted future stream of benefits. Yet, the main intuition that a firm with an *ex ante* advantage would have a greater chance of retaining (and thus a greater value of obtaining) incumbency would likely survive.

Our conclusions are limited, as always, to our specific setting. In particular, we consider only myopic consumers (auctioneers) in second-price, private-value settings, and firms that cannot commit to future prices. Nevertheless, the presence of incumbent advantage leads to substantially different predictions about the competitive effects of mergers than those obtained in the absence of incumbent advantage.

16

References

Appelman, M., Gorter, J., Lijesen, M., Onderstal, S. and Venniker, R.: 2003, Equal rules or equal opportunities? demystifying level playing field. CPB Netherlands Bureau for Economic Policy Analysis Document 34.

Arozamena, L. and Cantillon, E.: 2004, Investment incentives in procurement auctions, *Review of Economic Studies* **71**(1), 1–18.

Bagchi, A.: 2007, Selling licenses for a process innovation: The impact of the product market on the selling mechanism, *Canadian Journal of Economics* (forthcoming).

Baker, J. B.: 1997, Unilateral competitive effects theories in merger analysis, *Antitrust* **11**(2), 21–26.

Brannman, L. and Froeb, L.: 2000, Mergers, cartels, set-asides, and bidding preferences in asymmetric oral auctions, *The Review of Economics and Statistics* **82**(2), 283–290.

Brent, R. P.: 2002, *Algorithms for Minimization without Derivatives*, original volume 1973 edn, Mineola, New York: Dover.

Das Varma, G.: 2002, Standard auctions with identity-dependent externalities, *RAND Journal of Economics* **33**(4), 689–708.

Das Varma, G.: 2003, Bidding for a process innovation under alternate modes of competition, *International Journal of Industrial Organization* **21**(1), 15–37.

Elmaghraby, W.: 2007, Auctions within e-sourcing events, *Production and Operations Management* **16**(4), 409–422.

EU: 2004, Directive 2004/18/ec of the European Parliament and of the Council. 31 March 2004, Consolidated version 2008 January 1.

Farrell, J. and Klemperer, P.: 2007, Coordination and lock-in: Competition with switching costs and network effects, *in* M. Armstrong and R. Porter (eds), *Handbook of Industrial Organization*, Vol. 3, Elsevier, chapter 31, pp. 1967–2072.

Farrell, J. and Shapiro, C.: 1988, Dynamic competition with switching costs, *RAND Journal of Economics* **19**(1), 123–137.

Froeb, L., Tschantz, S. and Crooke, P.: 2001, Second-price auctions with mixtures of power-related distributions. Vanderbilt Working Paper.

Greenstein, S. M.: 1993, Did installed base given an incumbent any (measurable) advantages in federal computer procurement?, *RAND Journal of Economics* **24**(1), 19–39.

Jehiel, P. and Moldovanu, B.: 1996, Strategic nonparticipation, *RAND Journal of Economics* **27**(1), 84–98.

Jehiel, P. and Moldovanu, B.: 2000, Auctions with downstream interaction among buyers, *RAND Journal of Economics* **31**(4), 768–791.

Jehiel, P., Moldovanu, B. and Stacchetti, E.: 1996, How (not) to sell nuclear weapons, *American Economic Review* **86**(4), 814–29.

Jeitschko, T. D. and Wolfstetter, E.: 2002, Scale economies and the dynamics of recurring auctions, *Economic Inquiry* **40**(3), 403–414.

Jofre-Bonet, M. and Pesendorfer, M.: 2000, Bidding behavior in a repeated procurement auction: A summary, *European Economic Review* **44**(4–6), 1006–1020.

Kaplan, T. R. and Zamir, S.: 2007, Asymmetric auctions: Analytic solutions to the general uniform case. University of Exeter working paper.

Klemperer, P.: 1987, The competitiveness of markets with switching costs, *RAND Journal of Economics* **18**(1), 138–150.

Klemperer, P.: 1995, Competition when consumers have switching costs: An overview with applications to industrial organization, macroeconomics, and international trade, *Review of Economic Studies* **62**(4), 515–39.

Klemperer, P.: 2007, Bidding markets, *Journal of Competition Law and Economics* **3**(1), 1–47.

Lebrun, B.: 1999, First price auctions in the asymmetric n bidder case, *International Economic Review* **40**(1), 125–42.

Leufkens, K. and Peeters, R.: 2007, Synergies are a reason to prefer first-price auctions!, *Economics Letters* **97**(1), 64–69.

Lofaro, A. and Ridyard, D.: 2003, Switching costs and merger assessment – dont move the goalposts, *European Competition Law Review* **24**, 1–4.

Maskin, E. and Riley, J.: 2000, Asymmetric auctions, *Review of Economic Studies* **67**(3), 413–38.

McAfee, P. and McMillan, J.: 1988, *Incentives in Government Contracting*, Toronto, Canada: University of Toronto Press.

Plum, M.: 1992, Characterization and computation of nash-equilibria for auctions with incomplete information, *International Journal of Game Theory* **20**(4), 393–418.

Porter, R. H. and Zona, J. D.: 1993, Detection of bid rigging in procurement auctions, *Journal of Political Economy* **101**(3), 518–38.

Silva, D. G. D., Dunne, T. and Kosmopoulou, G.: 2003, An empirical analysis of entrant and incumbent bidding in road construction auctions, *Journal of Industrial Economics* **51**(3), 295–316.

Tang Sørensen, S.: 2006, Sequential auctions for stochastically equivalent complementary objects, *Economics Letters* **91**(3), 337–342.

Tschantz, S., Crooke, P. and Froeb, L.: 2000, Mergers in sealed vs. oral auctions, *International Journal of the Economics of Business* **7**(2), 201–213.

von der Fehr, N.-H. and Riis, C.: 2000, Option values in sequential markets. University of Oslo working paper.

Waehrer, K. and Perry, M. K.: 2003, The effects of mergers in open auction markets, *RAND Journal of Economics* **34**(2), 287–304.

Appendix

Proof of Proposition 1. The probability of bidder j winning given that i^* is the incumbent is given by:

$$p_j(i^*) = \frac{\alpha_j x_i}{\sum_{k=1}^n \alpha_k x_k} \qquad \text{where } \alpha_k = \begin{cases} e^{\Phi_k(i^*)} & k \neq i^* \\ e^{\Phi_k(i^*)+c_{inc}} & k = i^* \end{cases} \tag{21}$$

Substituting into (1), and dropping the dependence on time, provides

$$\Phi_j(i^*) = \left(v_j(j) - \frac{\sum_{k \neq j} p_k(i^*) v_j(k)}{\sum_{k \neq j} p_k(i^*)} \right) / (1+r) \tag{22}$$

$$= \frac{v_j(j) - \sum_{k=1}^n p_k(i^*) v_j(k)}{(1+r)(1 - p_j(i^*))} \tag{23}$$

$$\tag{24}$$

The expected surplus in the present period is given by:

$$\left(E[b_{-j}^{min}(i^*)|b_{-j}^{min}(i^*) > b_j(c_j, i^*)] - b_j(c_j, i^*) \right) p_j(i^*) = -\log(1 - p(j, i^*)) - \Phi_j(i^*) p_j(i^*) \tag{25}$$

Substituting into (3), we obtain

$$v_j(i^*) = -\log(1 - p_j(i^*)) - \Phi_j(i^*) p_j(i^*) + \sum_{k=1}^n p_k(i^*) v_j(k) / (1+r) \tag{26}$$

$$= -\log(1 - p_j(i^*)) - \Phi_j(i^*) p_j(i^*) - (1 - p_j(i^*)) \Phi_j(i^*) + v_j(j) / (1+r) \tag{27}$$

$$= -\log(1 - p_j(i^*)) - \Phi_j(i^*) + v_j(j) / (1+r) \tag{28}$$

and in particular

$$v_j(j) = -\log(1 - p_j(j)) - \Phi_j(j) + v_j(j) / (1+r) \tag{29}$$

$$= -\left(\log(1 - p_j(j)) + \Phi_j(j) \right) \frac{1+r}{r} \tag{30}$$

and therefore

$$v_j(i^*) = -\log(1 - p_j(i^*)) - \Phi_j(i^*) - \left(\log(1 - p_j(j)) + \Phi_j(j) \right) / r \tag{31}$$

Finally, substituting the value of $v_j(i^*)$ into (22),

$$\Phi_j(i^*) = \frac{v_j(j) - \sum_{k=1}^n p_k(i^*) v_j(k)}{(1+r)(1 - p_j(i^*))} \tag{32}$$

$$\tag{33}$$

$$= \frac{-\log(1 - p_j(j)) - \Phi_j(j) - \sum_{k=1}^n p_k(i^*) \left(-\log(1 - p_j(k)) - \Phi_j(k) \right)}{(1+r)(1 - p_j(i^*))} \qquad \square$$